Ohio's Learning Standards – Extended

Mathematics

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Ohio | Department of Education

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Introduction to Ohio's New Learning Standards - Extended: Mathematics

OVERVIEW

In February 2017, the state of Ohio adopted updated Ohio Learning Standards (OLS) for English language arts and math. Consequently, Ohio revised the Ohio Learning Standards-Extended (OLS-E) to be aligned with the OLS. The Ohio Department of Education collaborated with teams of educators and experts from around the state to do the work. These committees met multiple times to draft the new extensions. The Department then posted the drafts for public feedback and received hundreds of comments. After the public comment period, the committees considered the comments and implemented suggestions into the final version.

The OLS-E are specific statements of knowledge and skills linked to the expectations in the OLS. The purpose of the extensions is to build a bridge that provides grade level access for students with the most significant cognitive disabilities to the content of the OLS.

The Department developed the OLS-E specifically for students who qualify for and take the Alternate Assessment for Students with Significant Cognitive Disabilities (AASCD). These extensions do not replace the OLS for mathematics, they are aligned to them. Teachers may use the standards and extensions as a skill or knowledge progression when designing instruction and assessments. Using a standards progression provides flexible access from varied entry points and allows learners with the most significant cognitive disabilities to grow knowledge and skill across a modified curriculum that is linked to the grade-level standards. Educators can then use the link to grade-level targets or outcomes as comparison data in present levels of performance on an IEP. Because instruction and assessment should always consider the full range of extended standards and the links to the grade-level targets and outcomes, the OLS-E development committee designed this document so that the reader can reference the OLS and the extensions on the same page to easily see the progression.

While educators should use the extended standards to provide content that is directly aligned to the OLS for mathematics, they must also meet each child's individual education needs by incorporating other skills as necessary. Teachers should consider incorporating instruction with individual accommodations or supports students need to access the curriculum as well as non-academic skills needed for student success such as communication, self-determination, fine/gross motor, and social/emotional skills. Daily living and life skills are often represented within the standards as reading, speaking, listening, writing, and economics skills and should be taught and integrated with the extensions. Educational plans should also include any other additional skills necessary for each child's individual education needs and transition planning goals.

Educators can use the OLS-E to differentiate instruction for a wide range of students by using the extensions as entry points to the OLS, but they must do so with caution. Students who do not take Ohio's AASCD will take the general assessments aligned to the general standards. These extensions can provide entry points into the OLS. However, schools must remember that students who do not participate in the AASCD should transition to and will be assessed using the OLS.

Complexity Levels

The committee extended the Ohio Learning Standards to include three levels from "most complex" to "least complex". The complexity levels are comprised of three targets of varying difficulty aligned to each standard from the OLS. The extensions are codified individually for clear designation. The last letter in the extension code indicates the complexity level: "a" denotes the highest level of complexity, "b" denotes the middle complexity level and "c" denotes the lowest complexity level. In some instances, the committee tiered the verb of the extension to increase or decrease the complexity level. In other cases, the concept or skill within the OLS is tiered across the three complexity levels. It is important to move from left to right when reading the extensions. To determine where instruction should begin, educators should start with the general standard and then progress down through the complexity levels until finding the optimum starting point. It's important to note that no one should categorize students according to an extension level. Instead, instruction should build skills across the extensions to the highest level possible based on individual student strengths which may vary across standards. Ideally, when educators apply these extensions within each grade level one should see instruction occurring at all ranges of complexity. When citing standards for lesson and/or assessment design, educators should include the full complexity range, including the general standard. Citing standards in this way acknowledges a range of entry points and a range of learning progressions

*Additional mathematics standards for High School that represent complex numbers in the Ohio's New Learning Standards are not included in the extended standards since they are not considered common mathematics curriclulum for all college and career ready students.

Accessibility

The OLS-E do not specify individual accommodations or supports that may be necessary for students to access the curriculum. Teachers should consider the unique learning needs of each student and employ the Individualized Education Plan (IEP) designated supports and services when designing lessons. It is imperative that teachers provide specially designed instruction, assistive technology, accommodations and other supports needed to ensure full access to learning opportunities so that students can demonstrate their knowledge and skills.

Navigating the Ohio Learning Standard Extensions

The graphic illustrates the components of the Extensions:

Grade Level

Three levels of complexity

GRADE 3

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex			Least Complex
Operations and Algebraic Thinking			
Represent and solve problems involving multiplication and division.			
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that *a* x *b* means *a* groups of *b* objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each.)	**3.OA.1a** Represent products of whole numbers up to 10 × 10 using arrays, area models, or physical objects (whole numbers 0 through 10).	**3.OA.1b** Represent products with factors of 1s, 2s, 3s, 4s, 5s, and 10s using ar[illegible] area models, or physical objects (whole numbers [illegible] through 10).	**3.OA.1c** Represent products with factors of 1s, 2s, and 5s using arrays, area models, or [illegible]l objects (whole [illegible]s 1 through 10).
3.OA.2 Interpret whole-numb[illegible]ients of whole numbers, e.g., interpret 56 ÷ 8 as the [illegible] when 56 objects are pa[illegible] number of shares when [illegible] shares of 8 objects each. *For [illegible] which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**3.OA.2a** Represent quotients of single-digit whole numbers up to 100 (including 0).	**3.OA.2b** Represent quotients using arrays, area models, or other physical representations for whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100.	**3.OA.2c** Represent quotients using arrays, area models, or other physical representations for whole numbers factors of 1s, 2s, and 5s with products not exceeding 10, 20, and 50, respectively.

Domain

Cluster

Standards with Codification

Learning Standards for Grades 3 - 8

GRADE 3

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Operations and Algebraic Thinking			
Represent and solve problems involving multiplication and division.			
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that *a* x *b* means *a* groups of *b* objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each.)	**3.OA.1a** Represent products of whole numbers up to 10 × 10 using arrays, area models, or physical objects (whole numbers 0 through 10).	**3.OA.1b** Represent products with factors of 1s, 2s, 3s, 4s, 5s, and 10s using arrays, area models, or physical objects (whole numbers 1 through 10).	**3.OA.1c** Represent products with factors of 1s, 2s, and 5s using arrays, area models, or physical objects (whole numbers 1 through 10).
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**3.OA.2a** Represent quotients of single-digit whole numbers up to 100.	**3.OA.2b** Represent quotients using arrays, area models, or other physical representations for whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100.	**3.OA.2c** Represent quotients using arrays, area models, or other physical representations for whole numbers factors of 1s, 2s, and 5s with products not exceeding 10, 20, and 50, respectively.
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)	**3.OA.3a** Solve word problems with products and/or quotients of whole numbers using arrays, area models, or other physical representations (whole numbers factors of 0 through 10).	**3.OA.3b** Solve word problems with products of whole numbers 1s, 2s, 3s, 4s, 5s, and 10s using arrays, area models, or other physical objects (products not exceeding 100).	**3.OA.3c** Represent word problems with products of whole number factors of 1s, 2s, and 5s using arrays, area models or other physical representations (whole numbers 1 through 10).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x □ = 48; 5 = □ ÷ 3; 6 × 6 = □.*	**3.OA.4a** When given a physical or visual model representing a multiplication fact (whole number factors of 0 through 10 with products not exceeding 100) and a set of 3 answer choices, identify the unknown whole number.	**3.OA.4b** When given a physical or visual model representing a multiplication fact (whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100) and a set of 3 answer choices, identify the unknown whole number.	**3.OA.4c** Match a provided physical or visual model to one of three provided multiplication or division number sentences. AND When given a number sentence, identify the operations symbol for ÷, ×, and =.
Understand properties of multiplication and the relationship between multiplication and division.			
3.OA.5 Apply properties of operations as strategies to multiply and divide. *For example, if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative Property of Multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (Associative Property of Multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive Property).* Students need not use formal terms for these properties.	**3.OA.5a** Physically or visually solve multiplication or division number sentences (whole number factors of 0 through 10 with products not exceeding 100) using the commutative and/or distributive properties (e.g., solving 3 × 8 by adding 3 × 5 to 3 × 3).	**3.OA.5b** Physically or visually solve multiplication number sentences (whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100) using the commutative property.	**3.OA.5c** Physically or visually match multiplication number sentences using the commutative property.
3.OA.6 Understand division as an unknown factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*	**3.OA.6a** Understand that the inverse operation of division is multiplication. Can answer a multiplication question to solve for division (What times 3 equals 15?). Then solve the division problem (understand that 3 groups of 5 equals 15).	**3.OA.6b** Understand division as the inverse operation of multiplication by sorting objects or pictures into equal groups and matching multiplication/division problems (3 x 5 = 15 15 ÷ 3 = 5).	**3.OA.6c** Show division as sorting objects or pictures into equal groups.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←→			Least Complex
Multiply and divide within 100.			
3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8, or properties of operations. Limit to division without remainders. By the end of grade 3, know from memory all products of two one-digit numbers.	**3.OA.7a** Fluently know all products (whole number factors of 0 through 10) and their respective division problems.	**3.OA.7b** Fluently know all products for whole number factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 100.	**3.OA.7c** Solve multiplication number sentences for multiples of 1s, 2s, and 5s (whole numbers 1 through 10) using arrays, area models, or other physical representations.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.			
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.	**3.OA.8a** Represent a 2-step problem using an equation with a symbol (e.g., shape) standing for the unknown and solve.	**3.OA.8b** Identify the array, area model, or other physical representation that shows the solution of a 1-step number sentence from a word problem (excludes division).	**3.OA.8c** Identify the number sentence that correlates with a given 1-step word problem (excludes division).
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	**3.OA.9a** Identify and explain arithmetic patterns in a number chart or addition and multiplication tables.	**3.OA.9b** Identify arithmetic patterns in a number chart, or addition and multiplication tables.	**3.OA.9c** Use odd or even numbers to identify/make a pattern using repeated addition within a 100s chart.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←		→	Least Complex
Number and Operations in Base Ten			
Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.			
3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	**3.NBT.1a** Use place value understanding and a physical and/or visual representation to round multi-digit whole numbers to the nearest 10 or 100.	**3.NBT.1b** Identify a given number to the nearest 10s place when using number lines and/or number grids (e.g., 22 will round to 20).	**3.NBT.1c** Using a physical or visual representation for numbers 0 through 10, when shown two numbers, show which number is closer to 0 or 10 (e.g., shown 5 or 6, student is asked which number shown is closer to 10).
3.NBT.2 Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**3.NBT.2a** Add and subtract within 500 using strategies based on place value, and the relationship between addition and subtraction (no calculator).	**3.NBT.2b** Add and subtract within 100 using strategies based on place value, and the relationship between addition and subtraction (no calculator).	**3.NBT.2c** Add and subtract within 20 using strategies based on place value, and the relationship between addition and subtraction (no calculator, but could include concrete objects or number charts).
3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range of 10–90, e.g., 9 × 80, 5 × 60, using strategies based on place value and properties of operations.	**3.NBT.3a** Multiply one-digit whole numbers by multiples of 10 using visual and/or physical representation.	**3.NBT.3b** Multiply one-digit whole numbers by 10 (e.g., 3 × 10 = 30).	**3.NBT.3c** When shown a number sentence of one-digit whole number multiplied by 10, match the product to the number sentence when shown 2 possible products (e.g., 5 x 10 = 50 or 80).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷ Least Complex			
Number and Operations – Fractions			
Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.			
3.NF.1 Understand a fraction $^1/_b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $^a/_b$ as the quantity formed by a parts of size $^1/_b$.	**3.NF.1a** Match fractions with their model (limit to fractions with denominators of 2, 3, 4, 6, 8).	**3.NF.1b** Match fractions with their model (limit to $^1/_3$, $^2/_3$, $^1/_4$, $^1/_2$, *and* $^3/_4$).	**3.NF.1c** Identify a unit fraction ($^1/_4$ or $^1/_2$) as part of a whole when shown as a physical and/or visual representation.
3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. **a.** Represent a fraction $^1/_b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $^1/_b$ and that the endpoint of the part based at 0 locates the number $^1/_b$ on the number line. **b.** Represent a fraction $^a/_b$ (which may be greater than 1) on a number line diagram by marking off a lengths $^1/_b$ from 0. Recognize that the resulting interval has size $^a/_b$ and that its endpoint locates the number $^a/_b$ on the number line.	**3.NF.2a** Identify fractions on a number line marked in equal parts matching the fraction denominator (limit to fractions with denominators of 2, 3, 4, 6, 8).	**3.NF.2b** Identify fraction(s) on a number line marked in equal parts matching the fraction(s)' denominator (limit to denominators of 2, 3 and 4).	**3.NF.2c** Identify a fraction on a number line marked in equal parts matching the fraction denominator (limit to $^1/_2$ *and* $^1/_4$).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a.** Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. **b.** Recognize and generate simple equivalent fractions, e.g., $^{1}/_{2} = ^{2}/_{4}$, $^{4}/_{6} = ^{2}/_{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **c.** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = ^{3}/_{1}$*; recognize that* $^{6}/_{1} = 6$*; locate* $^{4}/_{4}$ *and 1 at the same point of a number line diagram.* **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**3.NF.3a** Use a visual fraction model to identify greater than, less than, and equal to when comparing 2 fractions.	**3.NF.3b** Use visual fraction models to identify equivalent fractions with denominators of 2, 4, 6, and 8.	**3.NF.3c** Identify equivalent fractions of $^{1}/_{2}$ and $^{1}/_{4}$ when represented with visual fraction models (e.g. matching model of $^{1}/_{2}$ and $^{2}/_{4}$ on a number line).
<u>Measurement and Data</u>			
Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.			
3.MD.1 Work with time and money. **a.** Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. **b.** Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ₵ symbol appropriately (not including decimal notation).	**3.MD.1a1** Tell time to the nearest 15 minutes on an analog clock. **3.MD.1a2** Name and/or identify equivalent combinations of coins and/or bills.	**3.MD.1b1** Tell time to the nearest 30 minutes on an analog clock. **3.MD.1b2** Identify, name, and state value for all coins and bills (coins: pennies, nickels, dimes, quarters; bills: $1, $5, $10, $20).	**3.MD.1c1** Tell time to the nearest hour on an analog clock. **3.MD.1c2** Identify and name all coins and bills.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷ Least Complex			
3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2, page 96.	**3.MD.2a** Solve 1-step word problems involving measures of liquid volumes and masses of objects using standard units of measure.	**3.MD.2b** Using models and drawings, measure and estimate liquid volumes and masses of objects using standard units of measure (e.g., measuring cup, scale).	**3.MD.2c** Select the appropriate tool to measure volume and mass (e.g., measuring cup, scale).
Represent and interpret data.			
3.MD.3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.*	**3.MD.3a** Create scaled bar (or picture) graph from given or collected data sets and interpret the graph, including solving 1-step (e.g., "how many more" "how many less" problems).	**3.MD.3b** Identify quantities from a picture or bar graph (e.g., in a class graph representing pets, represent 4 cats with 4 blocks or 4 cat pictures and 2 hamsters with 2 blocks or pictures).	**3.MD.3c** Sort data on a bar graph (e.g., weather–sunny, cloudy, rainy, snowy)
3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	**3.MD.4a** Measure objects using a ruler to the nearest fourth of an inch.	**3.MD.4b** Measure objects using a ruler to the nearest half inch.	**3.MD.4c** Measure objects using a ruler to the nearest inch.
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	**3.MD.5a** Recognize the number of units in a given surface area represents an array multiplication problem.	**3.MD.5b** Understand that an equal-sided square can represent 1 unit of measure and can be counted to determine the area of a plane figure.	**3.MD.5c** Understand that the term "area" is related to measurement of a surface.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	**3.MD.6a** Find the area of rectangles with whole-number side lengths by counting unit squares (limit area up to 40).	**3.MD.6b** Find the area of rectangles with whole-number side lengths by counting unit squares (limit to factors of 1s, 2s, 3s, 4s, 5s, and 10s with products not exceeding 30).	**3.MD.6c** Find the area of rectangles with whole-number side lengths by counting unit squares (limit factors of 1s, 2s, and 5s and areas up to 20).
3.MD.7 Relate area to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model). **d.** Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	**3.MD.7a** Given the side length measures for a rectangle, find the area (whole number factors with areas limited to 40).	**3.MD.7b** Given a visual model of a tiled rectangle, identify a number sentence (repeated addition or multiplication) that represents a solution for finding the area (whole number factors with areas limited to 30).	**3.MD.7c** Use tiling to cover the area of a square and count the tiles (unit squares) to find the area (whole number factors with areas limited to 20).
Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures			
3.MD.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	**3.MD.8a** Solve one-step measurement word problems involving shapes with the same area and different perimeters.	**3.MD.8b** Solve addition or subtraction measurement word problems involving perimeter.	**3.MD.8c** Solve addition measurement problems by finding the perimeter of a rectangle represented on a grid.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
<u>Geometry</u>			
Reason with shapes and their attributes.			
3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).	**3.G.1a** Sort quadrilaterals by the number of sides and/or the presence or absence of square corners (right angles) (limit quadrilaterals to rectangles, squares, and rhombuses).	**3.G.1b**. Sort polygons with up to 8 sides by the number of sides (Limit quadrilaterals to rectangles, squares and rhombuses).	**3.G.1c** Match objects in the environment to their two-dimensional shape based on the number of sides (e.g., match a stop sign in the real world to an octagon shape).
3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $^1/_4$ of the area of the shape.*	**3.G.2a** Partition rectangles into two, three, or four equal parts; identify a part as $^1/_2$, $^1/_4$, $^1/_3$.	**3.G.2b** Partition rectangles into two or four equal parts, identify the parts as "halves," "quarters," and whole.	**3.G.2c** Count the number of sections in a rectangle that has been divided into equal parts (limit to half and quarter).

GRADE 4

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵			⟶ Least Complex
Operations and Algebraic Thinking			
Use the four operations with whole numbers to solve problems.			
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**4.OA.1a** Solve a multiplicative comparison using a visual and/or physical representation (limited to multiples of 2s, 3s, 4s, 5s, 6s, and 10s).	**4.OA.1b** Solve a multiplicative comparison problem using linear or other physical representations (limit to whole number factors of 2s, 4s, 5s, and 10s).	**4.OA.1c** Solve a multiplicative comparison problem using linear models or other physical representations (limit to whole number factors of 2s and 5s).
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)	**4.OA.2a** Solve for the unknown product or quotient when given a word problem involving a multiplicative comparison (limit to whole number factors to 10 × 10).	**4.OA.2b** Given an array, area model, or other physical model that best represents a multiplicative comparison in a word problem, solve for the unknown product or quotient (limit to whole number factors of the 2s, 4s, 5s, and 10s).	**4.OA.2c** Identify the array, area model, or other physical representation that best represents a multiplicative comparison in a word problem (limit to multiples of 2s and 5s).
4.OA.3 Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**4.OA.3a** Determine the operation and correctly solve one-step word problems with remainders when given visual and/or physical representations (whole numbers within 1,000).	**4.OA.3b** Determine the operation(s) and correctly solve two-step word problems, without remainders, when given visual and/or physical representations (whole numbers; sums to 100).	**4.OA.3c** Solve a one-step word problem using a given visual and/or physical model (whole numbers; sums to 30; factors of 1s, 2s, 5s, and 10s).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
Gain familiarity with factors and multiples.			
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	**4.OA.4a** Using a multiplication table or other tool, identify the factor pairs for whole numbers in the range of 1-50.	**4.OA.4b** Match and/or sort factor pairs for numbers up to 50 with physical and/or visual representations (*for example, 50 matches: 1 × 50, 50 × 1, 10 × 5, 5 × 10, 2 × 25, and 25 × 2; 37 matches 37 × 1 and 1 × 37).*	**4.OA.4c** Match factor pairs for whole numbers up to 20 with physical and/or visual representations (*for example, 12 matches: 1 × 12 and 12 × 1, 3 × 4 and 4 × 3, and 6 × 2 and 2 × 6).*
Generate and analyze patterns.			
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**4.OA.5a** Given a rule for a pattern and its visual and/or physical representation, extend the pattern or identify or exclude objects or numbers that don't fit the rule of the pattern from physical and/or visual representations.	**4.OA.5b** Extend a shape or number pattern up to five terms given physical and/or visual representations.	**4.OA.5c** Extend a shape pattern two terms using a visual or physical representation (manipulatives).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ↔			Least Complex
Numbers and Operations in Base Ten			
Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.			
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.	**4.NBT.1a** Decompose multi-digit whole numbers by their place values and expanded form up to 100,000 with physical and/or visual representations (for example, 457: 4 hundreds, 5 tens, 7 ones; four hundred fifty-seven; 400 + 50 + 7).	**4.NBT.1b1** Given a whole number within the range of 1-999, decompose into place values of ones, tens and hundreds using physical and/or visual representations. AND **4.NBT.1b2** Given a whole number within the range of 1-999, identify the expanded form using physical and/or visual representations.	**4.NBT.1c** Given a whole number within the range of 1-99, identify the value in the ones place and/or the tens place using a place value chart or other visual/physical representation.
4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.	**4.NBT.2a1** Use place value knowledge to compare 2 numbers using >, =, and < symbols along with physical and/or visual representations (whole numbers 1-10,000). **4.NBT.2a2** Read and write numbers up to 10,000 in standard and expanded form.	**4.NBT.2b1** Use place value knowledge to compare 2 numbers using >, <, = symbols along with physical and/or visual representations (whole numbers 1-1000). **4.NBT.2b2** Given a number in standard form or word form, write the number in expanded form. *For example, 206 = 200 + 6* (whole numbers 1-1000).	**4.NBT.2c** Match the word form or standard form of two-digit whole numbers with physical and/or visual representations of objects and place values. *For example, "25" or the word "twenty-five" is matched to a set of 25 objects and/or 2 tens and 5 ones cubes* (whole numbers to 99).

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷ Least Complex			
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.	**4.NBT.3a** Round whole numbers to any place using physical and/or linear visual representations (whole numbers to 10,000, number lines).	**4.NBT.3b** Round whole numbers to the nearest 10 or 100 using physical and/or linear visual representations (whole numbers to 1,000, number lines).	**4.NBT.3c** Round two-digit whole numbers to the nearest 10 using a physical and/or linear visual representation (whole numbers to 99, number lines).
Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.			
4.NBT.4 Fluently add and subtract multi-digit whole numbers using a standard algorithm.	**4.NBT.4a** Add and subtract (with regrouping) 3-digit whole numbers using place value strategies and/or physical or visual representations (sums within 10,000).	**4.NBT.4b** Add and subtract up to two 3-digit whole numbers using place value strategies and/or physical or visual representations (including: adding two 2-digit whole numbers whose sums are less than 100 and may require regrouping; and adding two 3-digit numbers without regrouping whose sums are less than 1000; subtraction of two 2-digit or two 3-digit numbers without regrouping).	**4.NBT.4c** Add and subtract whole numbers using place value strategies and/or physical or visual representations. (Including sums of three one-digit whole numbers within 30, sums of 1-digit and 2-digit whole numbers with regrouping allowed in ones, and sums of two 2-digit whole numbers whose sums are within 100 without regrouping; subtraction of up to two 2-digit numbers without regrouping whose sums are within 100).
4.NBT.5 Multiply a whole number of up to four digits by a 1-digit whole number, and multiply two 2-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**4.NBT.5a** Multiply a 2-digit whole number by a 2- digit whole number, using strategies based on place value and the properties of operations with arrays, area models, or other visual representations (products within 10,000).	**4.NBT.5b** Multiply a 3-digit whole number by a 1-digit whole number, using strategies based on place value and the properties of operations along with arrays, area models, or other physical representations (products within 1,000).	**4.NBT.5c** Multiply a multiple of 10 by a 1-digit whole number using arrays, area models, or other physical representations.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟵⟶			Least Complex
4.NBT.6 Find whole-number quotients and remainders with up to 4-digit dividends and 1-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**4.NBT.6a** Divide a 3-digit whole number by a 1-digit whole number using strategies based on place value, relationship between multiplication and division and the properties of operations using arrays, area models or other physical/visual representations (whole numbers answers with no remainders).	**4.NBT.6b** Divide multiples of 10 up to 90 by a 1-digit whole number using strategies based on place value, the relationship between multiplication and division, and the properties of operations using arrays, area models, or other physical/visual representations (whole numbers to 90 with no remainders).	**4.NBT.6c** Determine whether a number is divisible by 2, 5, or 10 for numbers up to 50 using physical and/or visual representations (whole numbers to 100).
Numbers and Operations – Fractions			
Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.			
4.NF.1 Explain why a fraction $^a/_b$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	**4.NF.1a** Write or model equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical/visual models. (fraction strips, number lines, area models).	**4.NF.1b** Identify equivalent fractions for denominators 2, 3, 4, 5, 6, 8, 10 when given rectangular fraction models or other physical visual models, *for example, matching model of* $^1/_2$ *and* $^2/_4$.	**4.NF.1c** Match fractions with their fraction model or other physical/visual models (limit to $^1/_3$, $^2/_3$, $^1/_4$, $^1/_2$, *and* $^3/_4$).
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^1/_2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**4.NF.2a** Compare two fractions using models and >, =, and < symbols (limit denominators to 2, 3, 4, 5, 6, 8, 10).	**4.NF.2b** Using a given model, compare two fractions to identify which is "greater than", "less than", or "equal to" (limit fractions to unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10).	**4.NF.2c** Determine which fraction is larger or smaller given pairs of fractions and their models (limit to $^1/_3$, $^2/_3$, $^1/_4$, $^1/_2$, *and* $^3/_4$).

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ←		→	Least Complex
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)			
4.NF.3 Understand a fraction $^{a}/_{b}$ with $a > 1$ as a sum of fractions $^{1}/_{b}$. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $^{3}/_{8} = ^{1}/_{8} + ^{1}/_{8} + ^{1}/_{8}$; $^{3}/_{8} = ^{1}/_{8} + ^{2}/_{8}$; $2\,^{1}/_{8} = 1 + 1 + ^{1}/_{8} = ^{8}/_{8} + ^{8}/_{8} + ^{1}/_{8}$. **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**4.NF.3a** Using physical and/or visual representation and fractions with denominators of 3, 4, 5, 6, 8, 10 and 100). **a.** Add and subtract fractions with like denominators. **b.** Decompose a mixed number into sums of fractions, *for example,* $1\frac{3}{4} = ^{4}/_{4} + ^{1}/_{4} + ^{1}/_{4} + ^{1}/_{4}$. **c.** Add and subtract fractions with like denominators *for example,* $1\,^{1}/_{3} + 1\,^{1}/_{3} = 2\,^{2}/_{3}$ *or* $1\,^{2}/_{3} - ^{1}/_{3} = 1\,^{1}/_{3}$ (including mixed numbers). **d.** Solve one-step real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole).	**4.NF.3b** Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10: **a.** Add and subtract fractions with like denominators. **b.** Decompose a mixed number into sums of unit fractions, *for example,* $1\,^{3}/_{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. **c.** Add and subtract a mixed number with a fraction of the same denominator. **d.** Solve real-world problems involving addition or subtraction of fractions with like denominators (referring to the same whole).	**4.NF.3c** Using physical models or visual representations and fractions with denominators of 2, 3 and 4 **a.** Add fractions with like denominators. **b.** Decompose a fraction into sums of unit fractions, *for example,* $^{3}/_{4} = ^{1}/_{4} + ^{1}/_{4} + ^{1}/_{4}$. **c.** Know that one whole (partitioned into equal-sized parts) equals the sum of all its equal parts. *For example, 1 whole = 4 fourths* ($^{4}/_{4}$) *or 3 thirds* ($^{3}/_{3}$). **d.** Solve one-step real-world problems involving addition of unit fractions (referring to the same whole).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←			→ Least Complex
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **a.** Understand a fraction $^a/_b$ as a multiple of $^1/_b$. *For example, use a visual fraction model to represent $^5/_4$ as the product 5 × ($^1/_4$), recording the conclusion by the equation $^5/_4$ = 5 × ($^1/_4$) or $^5/_4$ = ($^1/_4$) + ($^1/_4$) + ($^1/_4$) + ($^1/_4$) + ($^1/_4$).* **b.** Understand a multiple of $^a/_b$ as a multiple of $^1/_b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × ($^2/_5$) as 6 × ($^1/_5$), recognizing this product as $^6/_5$. (In general, n × ($^a/_b$) = $^{(n \times a)}/_b$.)* **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $^3/_8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	***4.NF.4a*** *Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, three friends each ate $^2/_8$ of a pizza. How much pizza did they eat? (limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers).*	**4.NF.4b** Identify equivalent number sentences for fractions expressed as sums of unit fractions and products of a one-digit whole number multiplied by the same unit fraction, for example, $^3/_4$ = $^1/_4$ + $^1/_4$ + $^1/_4$ *matches* 3 × $^1/_4$ (limit to fractions with denominators of 2, 3, 4, 5, 6, 8 and 10; physical or visual fraction models may be used).	**4.NF.4c** Match a whole number with its given equivalent fraction using physical and/or visual representations for mathematical and word problems. (*For example, 4 = $^4/_1$).*
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $^3/_{10}$ as $^{30}/_{100}$, and add* $^3/_{10}$ + $^4/_{100}$ = $^{34}/_{100}$. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.	**4.NF.5a** Identify a fraction with a denominator of 10 as an equivalent fraction with denominator 100. *For example,* $^{30}/_{10}$ = $^3/_{10}$).	**4.NF.5b** Match a fraction with a denominator of 10 with its equivalent physical or visual model.	**4.NF.5c** Match a fraction with a denominator of 100 with its equivalent physical or visual model.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷			Least Complex
4.NF.6 Use decimal notation for fractions with denominators of 10 or 100. *For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	**4.NF.6a** Rewrite a fraction with a denominator of 100 as a decimal using place value visual and/or physical representations. *For example, rewrite $^{62}/_{100}$ as 0.62.*	**4.NF.6b** Rewrite a fraction with a denominator of 10 as a decimal. *For example, rewrite $^{2}/_{10}$ as 0.2 using place value, physical and/or visual representations.*	**4.NF.6c** Match a collection of pennies or dimes to the visual model of the decimal. AND Select the decimal that represents a visual and/or physical model for a collection of pennies or dimes.
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**4.NF.7a** Compare two decimals using place value models and the <, >, and = symbols (limit to tenths with tenths and hundredths to hundredths, includes whole numbers to tens).	**4.NF.7b** Compare two decimals using place value models and the <, >, and = symbols (limit to tenths with tenths and hundredths to hundredths, no whole numbers).	**4.NF.7c** Identify the tenths and hundredths place on a place value chart and in a given decimal using physical or visual representations.
Measurement and Data			
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.			
4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, and 3 and 300.*	**4.MD.1a** Convert between km and m, m and cm, kg and g using place value charts or other physical/visual representations.	**4.MD.1b** Identify whether a measurement is "more than," "less than," or "same as" another metric measurement using a place value chart or other physical/visual models. *For example, 1.5 kg is larger than 500 g.*	**4.MD.1c** Determine the best metric unit to measure a specific real-world item (metric units include centimeters and meters, kilograms and grams, liters).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵	⟶		Least Complex
4.MD.2 Solve real-world problems involving money, time, and metric measurement. **a.** Using models, add and subtract money and express the answer in decimal notation. **b.** Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes. **c.** Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.	**4.MD.2a1** Solve real-world problems involving addition or subtraction of coins and bills using visual and/or physical representations (limit amounts to less than $100). **4.MD.2a2** Solve word problems involving addition and subtraction of time intervals in 15 minutes with visual and/or physical representations. **4.MD.2a3** Solve real-world problems involving mass or volume by selecting appropriate operations with physical and/or visual representations.	**4.MD.2b1** Solve real-world problems with addition of collections of coins or bills with visual and/or physical representations (limit amounts to less than $50). **4.MD.2b2** Solve word problems involving addition of time intervals of 30 minutes with visual and/or physical representations. **4.MD.2b3** Solve real-world problems by measuring liquid volumes and masses of objects using standard units of measure with physical and/or visual representations.	**4.MD.2c**1 Identify the value of all coins. Find the total of a collection of all pennies or all dimes or all nickels. **4.MD.2c2** Solve word problems involving addition of time intervals of one hour with visual and/or physical representation. **4.MD.2c3** Solve real-world problems by selecting the appropriate tool to measure metric volume or mass with visual and physical representations.
4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.*	**4.MD.3a1** Given a rectangle with side lengths marked, find area (limit to whole number measurements). **4.MD.3a2** Solve real-world problems involving perimeter. Find the perimeter or given the perimeter, find the missing side length (limit to whole numbers).	**4.MD.3b1** Find the area of rectangles by counting unit squares and understand that a square with a side length of 1 unit is called a "unit square." **4.MD.3b2** Find the perimeter of rectangles drawn on grid paper (whole numbers only).	**4.MD.3c1** Find the area of rectangles with whole-number side lengths by counting unit squares. **4.MD.3c2** Find the perimeter of rectangles by counting the number of unit squares that fit around the shape.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex	←	→	Least Complex
Represent and interpret data.			
4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade.	**4.MD.4a** Interpret data from a given line, picture, or bar graph to solve a multi-step problem (limit to whole numbers).	**4.MD.4b** Interpret data represented in a graph by solving one-step "how many more" and "how many less" problems (limit to whole numbers).	**4.MD.4c** Given a bar or picture graph, build a graph based on student sorted data. *For example, votes for 4 different candidates or weather types, or occurrences of event or behavior.*
Geometric measurement: Understand concepts of angle and measure angles.			
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **a.** Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $^{1}/_{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. **b.** Understand an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	**4.MD.5a** Label the parts of an angle (two rays, common endpoint) and identify, 45-, 90-, 180-, and 360-degree angles.	**4.MD.5b** Match words to the parts of an angle (two rays, common endpoint).	**4.MD.5c** Identify an angle in a given shape.
4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	**4.MD.6a** Measure 45-, 90-, and 180-degree angles.	**4.MD.6b** Match measures and diagrams of 45-, 90-, and 180-degree angles.	**4.MD.6c** Given two measuring tools, select a protractor as a tool to measure and sketch angles.
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**4.MD.7a** Solve measurement problems involving angles using addition and subtraction.	**4.MD.7b** Solve measurement problems involving angles using addition.	**4.MD.7c** Solve addition measurement problems involving angles to show that two smaller angles make a larger angle.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
Geometry			
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.			
4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	**4.G.1a** Match word to corresponding picture of perpendicular and parallel lines and angles (right, acute, obtuse).	**4.G.1b** Match word to corresponding picture of points, lines, line segments rays, and angles.	**4.G.1c** Identify a point, line, and a line segment.
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.	**4.G.2a** Label a two-dimensional shape to show understanding of parallel or perpendicular lines.	**4.G.2b** Sort two-dimensional shapes based on presence of parallel and/or perpendicular lines.	**4.G.2c** Match two-dimensional shapes in the environment based on parallel lines.

GRADE 5

<table>
<tr><th>Learning Standard</th><th>Complexity a</th><th>Complexity b</th><th>Complexity c</th></tr>
<tr><td colspan="2">Most Complex ⟷</td><td colspan="2">Least Complex</td></tr>
<tr><td colspan="4"><u>Operations and Algebraic Thinking</u></td></tr>
<tr><td colspan="4">Write and interpret numerical expressions.</td></tr>
<tr><td>5.OA.1 Use parentheses in numerical expressions and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.</td><td>5.OA.1a Solve a two-step expression involving addition and subtraction and parentheses. For example, 5 + (6 — 3) = 5 + 3 = 8 (limit to one-digit whole numbers).</td><td>5.OA.1a Identify the first step in solving a two-step problem involving addition and subtraction.</td><td>5.OA.1c Identify parentheses as a marker of a group in a number sentence.</td></tr>
<tr><td>5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.</td><td>5.OA.2a Write a two-step numerical expression when given a number sentence; do not calculate.</td><td>5.OA.2b Match a two-step number sentence with its expression. For example, add 2 and 2 then subtract 1 matches (2 + 2) —1.</td><td>5.OA.2c Match a one-step number sentences with its expression/ For example, 2 plus 2 = 2 + 2.</td></tr>
<tr><td colspan="4">Analyze patterns and relationships.</td></tr>
<tr><td>5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</td><td>5.OA.3a Given a rule for a numerical pattern and a two-column table missing several terms, complete the table and graph in the first quadrant.
Add 4<table><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>8</td></tr><tr><td>5</td><td></td></tr></table></td><td>5.OA.3b Plot 2 ordered pairs in the first quadrant of a coordinate grid.</td><td>5.OA.3c Given a coordinate grid marked with locations of familiar locations (school, home, library, grocery), identify the location for an ordered pair.</td></tr>
</table>

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
Numbers and Operations in Base Ten			
Understand the place value system.			
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^{1}/_{10}$ of what it represents in the place to its left.	**5.NBT.1a** Decompose multi-digit whole numbers by their place values and expanded form to show understanding that a digit in one place represents 10 times as much as it represents in the place to its right.	**5.NBT.1b** Decompose multi-digit whole numbers by their place values and expanded form up to 100,000 with physical and/or visual representations. *For example, 457: 4 hundreds, 5 tens, 7 ones; four hundred fifty-seven; 400 + 50 + 7).*	**5.NBT.1c1** Given a whole number within the range of 1-999, decompose into place values of ones, tens, and hundreds using physical and/or visual representation. AND **5.NBT.1c2**Given a whole number within the range of 1-999, identify the expanded form using physical and/or visual representations.
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**5.NBT.2a** Identify the product of a two-digit decimal and a power of 10 (limit decimals to tenths and powers of 10 to 10^3).	**5.NBT.2b** Identify the power of 10 written as a base and an exponent for its multiplication sentence. *For example,* $10^? = 10$; $10^? = 10 \times 10$; $10^? = 10 \times 10 \times 10$. Identify the product of a 2-digit whole number and a power of 10 (limit powers of 10 to 1,000 or 10^3).	**5.NBT.2c** Identify the number of zeros in a product of a one-digit whole number and 10 or 100 using place value strategies or physical representations.
5.NBT.3 Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 $= 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (^{1}/_{10}) + 9 \times (^{1}/_{100}) + 2 \times (^{1}/_{1000})$. **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**5.NBT.3a** Compare two decimal numerals written to the hundredths place using >, = and < symbols with visual and/or physical representations.	**5.NBT.3b** Compare two decimal models to the tenths place using >, = and < symbols with visual and/or physical representations.	**5.NBT.3c** Match visual or physical representations or models of tenths and determine which is "more than", "same as", or "Less than".

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷ Least Complex			
5.NBT.4 Use place value understanding to round decimals to any place, millions through hundredths.	**5.NBT.4a** Round decimals to a given place using a number line or other physical/visual representation (limit rounding to the nearest tenth, one, ten or hundred).	**5.NBT.4b** Round decimals with a value in the tenths place to the nearest whole number using a number line or other physical/visual representation.	**5.NBT.4c** Identify whether a decimal is closer to 0 or 1 using physical and/or visual representations, including money (limit decimal place values to hundredths).
Perform operations with multi-digit whole numbers and with decimals to hundredths.			
5.NBT.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	**5.NBT.5a** Multiply a 3-digit whole number by a 2-digit whole number with visual and/or physical representations.	**5.NBT.5b** Multiply 2-digit by 2-digit whole numbers with physical and/or visual representations.	**5.NBT.5c** Multiply two 2-digit whole numbers, where one factor is a multiple of 10, using models with physical or/and visual representations
5.NBT.6 Find whole-number quotients of whole numbers with up to 4-digit dividends and 2-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**5.NBT.6a** Divide whole numbers using strategies based on place value, the relationship between multiplication and division, the properties of operations and physical and/or visual representations (limit to 4-digit whole number by a 1-digit whole number, no remainders).	**5.NBT.6b** Divide whole numbers using strategies based on place value, the relationship between multiplication and division, the properties of operations and physical and/or visual representations (3-digit by 1-digit whole numbers with no zeroes in the tens place, no remainders).	**5.NBT.6c** Divide a 2-digit whole number by a 1-digit whole number using strategies based on place value, the relationship between multiplication and division, the properties of operations and using physical and/or visual representations (products limited to 100, no remainders).
5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers (whole numbers through the hundreds place and decimals through the hundredths place). **b.** Multiply whole numbers by decimals (whole numbers	**5.NBT.7a1** Add and subtract decimals to hundredths with physical and/or visual representations with word problems. AND **5.NBT.7a2** Multiply and divide up to a 3-digit whole number by a 2-digit number with a digit in the tenths place value,	**5.NBT.7b1** Add and subtract decimals to hundredths using physical and/or visual representations with and without word problems. AND **5.NBT.7b2** Multiply or divide up to a 3-digit whole number by a decimal to the tenths place with physical and/or	**5.NBT.7c1** Add and subtract decimals to tenths using physical and/or visual representations with word problems related to money. AND **5.NBT.7c2** Multiply a decimal to the tenths place by a single digit whole number with physical and/or visual

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place, using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*	with physical and/or visual representations, with or without word problem. *For example, 312 x 1.2.*	visual representations, with or without word problem. *For example, 312 x 0.3.*	representations, with or without word problem. *For example, 7 x 0.1.*
Numbers and Operations – Fractions			
Use equivalent fractions as a strategy to add and subtract fractions (fractions need not be simplified.).			
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $^2/_3 + ^5/_4 = ^8/_{12} + ^{15}/_{12} = ^{23}/_{12}$.	**5.NF.1a** Add and subtract fractions with fraction models or other physical and/or visual representations. *For example,* $^2/_4 + ^2/_8 = ^6/_8$. Limit denominators to multiple pairs of 3 and 6, 2 and 4, 4 and 8, 5 and 10, or 10 and 100, may include mixed numbers.	**5.NF.1b** Add and subtract fractions with same denominator fraction models or other physical and/or visual representations (limit denominators to 2, 3, 4, 5, 6, 8, and 10).	**5.NF.1c** Add and subtract fractions with the same denominator using fraction models or other physical and/or visual representations (limit denominators to 2, 3, 4, and 10).
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $^2/_5 + ^1/_2 = ^3/_7$, *by observing that* $^3/_7 < ^1/_2$.	**5.NF.2a** Solve addition and subtraction fraction word problems with fraction models or other physical and/or visual representations. Limit denominators to multiple pairs of 3 and 6, 2 and 4, 4 and 8, 5 and 10, or 10 and 100, may include mixed numbers.	**5.NF.2b** Solve addition and subtraction word problems involving fractions with like denominators with fraction models or other physical and/or visual representations. Limit denominators to 2, 3, 4, 5, 6, 8 and 10.	**5.NF.2c** Solve addition and subtraction word problems involving fractions with like denominators with fraction models or other physical and/or visual representations. Limit denominators to 2, 3, 4, and 10.
Apply and extend previous understandings of multiplication and division to multiply and divide fractions (fractions need not be simplified)			
5.NF.3 Interpret a fraction as division of the numerator by the denominator ($^a/_b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction	**5.NF.3a** Given fraction models or other physical/visual representations with or	**5.NF.3b** Given fraction models or other physical/visual representations with or	**5.NF.3c** Match a fraction to its division problem shown with a fraction model or other physical or visual

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷			Least Complex
models or equations to represent the problem. *For example, interpret $^3/_4$ as the result of dividing 3 by 4, noting that $^3/_4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size $^3/_4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	without a familiar context and limited to whole numbers up to 50, identify the number sentence when represented as a fraction. *For example, 48 pounds of nuts are shared by 6 children and $^{48}/_6$.* Solve the number sentence from above. *For example, 48 pounds of nuts shared by 6 children equals 8 (pounds of nuts for each child).*	without a familiar context and limited to whole numbers up to 20 with no remainders, identify the number sentence when represented as a fraction. *For example, 4 children share 3 candy bars matches (3 candy bars divided by 4) ¾, or 12 pounds of nuts shared by 6 children equals $^{12}/_6$.*	representation (limit fractions to denominators of 2, 3, 4, and 10).
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product $(^a/_b) \times q$ as a part of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(^2/_3) \times 4 = {}^8/_3$, and create a story context for this equation. Do the same with $(^2/_3) \times (^4/_5) = {}^8/_{15}$. (In general, $(^a/_b) \times (^c/_d) = {}^{ac}/_{bd}$.)* **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	**5.NF.4a** Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. For example, *3 friends each ate $^2/_8$ of a pizza. How much pizza did they eat?* (Limit to fractions with denominators of 2, 3, 4, 6, and 8; may involve mixed numbers.)	**5.NF.4b** Solve real-world problems involving multiplying a fraction by a whole number up to 10 using visual fraction models. *For example, 3 friends each ate $^2/_8$ of a pizza. How much pizza did they eat?* (Limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers.)	**5.NF.4c** Identify equivalent number sentences for fractions expressed as sums of unit fractions and products of a 1-digit whole number multiplied by the same unit fraction. *For example, $^3/_4 = {}^1/_4 + {}^1/_4 + {}^1/_4$ matches $3 \times {}^1/_4$.* (Limit to fractions with denominators of 2, 3, 4, 5, 6, 8 and 10; physical or visual fraction models may be used.)
5.NF.5 Interpret multiplication as scaling (resizing). **a.** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given	**5.NF.5a** Match a multiplication problem involving a fraction multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual and/or	**5.NF.5b** Match a multiplication problem involving a fraction multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual	**5.NF.5c** Match a multiplication problem using $^1/_2$ multiplied by a whole number to its represented picture to show knowledge that product will be smaller than whole number in problem with visual and/or

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷			Least Complex
number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^a/_b = {}^{(n\times a)}/_{(n\times b)}$ to the effect of multiplying $^a/_b$ by 1.	physical representations (limit to fractions with denominators of 2, 3, 4, 6, and 8; may involve mixed numbers.) Compare the product to the factors using <, >, = symbols.	and/or physical representations (limit to fractions with denominators of 2, 3, 4, 6, and 8; no mixed numbers). Compare the product to the factors using <, >, = symbols.	physical representations. *For example, 3 x $^1/_2$ = 1 $^1/_2$.* Compare the product to the factors.
5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	**5.NF.6a** Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10, solve real-world problems involving multiplication of fractions and mixed numbers.	**5.NF.6b** Using physical models or visual representations of fractions with denominators of 2, 3, 4, 5, 6, 8 and 10, solve real-world problems involving multiplication of fractions and whole numbers.	**5.NF.6c** Match the real-world word problem to the number sentence that involves multiplying a fraction by a whole number with physical and/or visual representation.
5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for ($^1/_3$) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ($^1/_3$) ÷ 4 = ($^1/_{12}$) because ($^1/_{12}$) × 4 = ($^1/_3$).* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ ($^1/_5$), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ ($^1/_5$) = 20 because 20 × ($^1/_5$) = 4.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by	**5.NF.7a** Solve real-world problems involving division of a fraction by a whole number using visual and/or physical representations (limit to denominators of 2, 3, 4, 5, 6, 8, and 10).	**5.NF.7b** Given a real-world context and its equation involving division of a fraction by a whole number, find the missing factor (limit numbers to 5 or less).	**5.NF.7c** Match the real-world word problem and its number sentence involving division of a fraction by a whole number to the physical and/or visual representation.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $^1/_2$ a pound of chocolate equally? How many $^1/_3$ cup servings are in 2 cups of raisins?*			
Measurement and Data			
Convert like measurement units within a given measurement system.			
5.MD.1 Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.	**5.MD.1a** Convert, within 1-step conversion, basic units of measure for cups to pints, quarts to gallons, and inches to feet, with visual and/or physical representations to solve real-world problems.	**5.MD.1b** Convert basic units of measure: hours to minutes, gallons to quarts, and yards to feet using physical and/or visual representations.	**5.MD.1c** Match physical or visual representations of measurement tools and units (time – clock, liquid – teaspoons, cups, or gallons, ruler – feet or yards).
Represent and interpret data.			
5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $^1/_2$, $^1/_4$, $^1/_8$, or decimals.	**5.MD.2a** Create a line plot, bar graph, or pictograph from a given or collected data set with measurements in fractions or decimals (fourths, halves, or tenths).	**5.MD.2b** Create a scaled graph – picture, bar, or line graph –from given or collected data sets, and interpret the graph, including solving 1- step "how many more" and "how many less" problems.	**5.MD.2c** Answer questions that interpret data represented in a picture, bar, or line graph by solving one-step "how many more" and "how many less" problems.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.			
5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a**. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b**. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	**5.MD.3a** Identify and measure the length, width, and height of a cube or rectangular prism to show understanding of the concept of volume (with physical or visual representations).	**5.MD.3b** Use a tool to fill a cube and/or rectangular prism to show understanding of volume.	**5.MD.3c** Match solid figure to its 2-dimensional shape counterpart with visual and/or physical representations.
5.MD.4 Measure volume by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	**5.MD.4a** Measure the volume of cubes and/or rectangular prisms by counting unit cubes.	**5.MD.4b** Count and write the volume of a cube by counting physical unit cubes.	**5.MD.4c** When given a model, student will build a cube or rectangular prism from unit cubes to show understanding of volume.
5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the Associative Property of Multiplication. **b.** Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.	**5.MD.5a** Find the volume when given a physical representation of a cube and/or rectangular prism in a real-world word problem.	**5.MD.5b** Label and measure the length, width, and height of a cube and/or rectangular prism in a real-world word problem with physical representations.	**5.MD.5c** Label the length, width, and height of a rectangular prism from a real-world word problem with physical representations.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex			Least Complex
Geometry			
Graph points on the coordinate plane to solve real-world and mathematical problems.			
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. When given a whole number ordered pair, student will plot the ordered pair using knowledge of the x-axis and y-axis.	**5.G.1a** When given a whole number ordered pair, student will plot the ordered pair using knowledge of the x-axis and y-axis.	**5.G.1b** Write the ordered pair for a plotted point in quadrant I of a coordinate plane.	**5.G.1c** Label the x-axis and the y-axis on quadrant I on a coordinate plane. Match a plotted point in quadrant I with its ordered pair.
5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**5.G.2a** Graph a point on a map and/or coordinate grid and use the information to solve real-world word problems.	**5.G.2b** Use the information on a graph to solve real-world word problems.	**5.G.2c** Identify a point on a map and/or coordinate grid.
Classify two-dimensional figures into categories based on their properties.			
5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).	**5.G.3a** Identify triangles (right, scalene, and isosceles) and the size of their angles.	**5.G.3b** Sort triangles by angle measures or side lengths when given physical or visual representations.	**5.G.3c** Sort triangles by the presence or absence of right angles (physical and/or visual representations given).
5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.	**5.G.4a** Identify quadrilaterals with perpendicular or parallel sides, right angles, or side lengths.	**5.G.4b** Sort quadrilaterals by given criteria (perpendicular sides, equal side lengths) using physical and/or visual representations.	**5.G.4c** Given polygons with three to eight sides, identify the quadrilaterals.

GRADE 6

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟵	⟷	⟷	⟶ Least Complex
Ratio and Proportional Relationships			
Understand ratio concepts and use ratio reasoning to solve problems.			
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote that candidate A received, candidate C received nearly three votes."*	**6.RP.1a** Given a visual model, represent two quantities as a ratio using whole numbers (e.g., In an image of 2 bananas and 3 oranges, what is the ratio of bananas to oranges?).	**6.RP.1b** Given a visual model or manipulative, identify ratios involving whole numbers (e.g., The ratio of bananas to oranges is 2 to 3. Which shows the correct ratio of bananas to oranges?).	**6.RP.1c** Given a manipulative, identify the units to be compared (e.g., Two bananas and three oranges are displayed. What two units are being compared?).
6.RP.2 Understand the concept of a unit rate $^{a}/_{b}$ associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $^{3}/_{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*	**6.RP.2a** Solve problems involving unit rates (e.g., If it took 2 hours to mow 6 lawns, how many lawns could be mowed in 8 hours at the same rate? At what rate were lawns being mowed?).	**6.RP.2b** Solve for a unit rate (e.g., It took James 2 hours to drive 100 miles. How fast did he drive per one hour?).	**6.RP.2c** Identify a unit rate in a word problem (e.g., James drives 65 miles per hour on the highway. How many miles does James drive in one hour?).

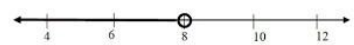
Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex			Least Complex
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **c.** Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $^{30}/_{100}$ times the quantity; solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**6.RP.3a1** Find a missing value in a ratio table. Students may use manipulatives to find the answer. AND **6.RP.3a2** Find the percent of a number using a model.	**6.RP.3b1** Build equal ratios with manipulatives and record information in a table. AND **6.RP.3b2** Find the 10%, 20%, and 30% of a number using a model.	**6.RP.3c1** Build equal ratios with manipulatives. AND **6.RP.3c2** Identify or represent a percent as a rate per 100 when given a model of 100 units (e.g., Mike gave away 20 of his 100 marbles. What percent of the marbles did he give away?).
The Number System			
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.			
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(^2/_3) \div (^3/_4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(^2/_3) \div (^3/_4) = {^8/_9}$ because $^3/_4$ of $^8/_9$ is $^2/_3$. (In general, $(^a/_b) \div (^c/_d) = {^{ad}/_{bc}}$.) How much chocolate will each person get if 3 people share $^1/_2$ pound of chocolate equally? How many $^3/_4$ cup servings are in $^2/_3$ of a cup of yogurt? How wide is a rectangular strip of land with length $^3/_4$ mi and area $^1/_2$ square mi?*	**6.NS.1a** Use visual models to show the relationship between the multiplication and division of fractions.	**6.NS.1b** Recognize that dividing a whole number by a fraction is separating the whole into the required fractional parts and counting how many parts are in the total (e.g., Given one yard of fabric divided into pieces that are $^2/_3$ of a yard, how many pieces will there be? Use a model of solve.).	**6.NS.1c** Recognize a fraction as the division of the numerator by the denominator using unit fractions (e.g., Use a model to show that $^1/_4$ means dividing a whole into 4 equal parts).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←			→ Least Complex
Compute fluently with multi-digit numbers and find common factors and multiples.			
6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.	**6.NS.2a** Divide multi-digit whole numbers (up to 3-digit numbers) by 1- or 2-digit numbers in problems with and without remainders.	**6.NS.2b** Divide a 2-digit number up to 100 by a 1-digit number with and without remainders using models.	**6.NS.2c** Divide a 2-digit whole number up to 20 by a 1-digit whole number without remainder using models.
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	**6.NS.3a** Add, subtract, and multiply multi-digit decimals using place value models.	**6.NS.3b** Add and subtract multi-digit decimals using place value models.	**6.NS.3c** Add decimals to the tenths place using place value models.
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100, and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	**6.NS.4a1** Identify the greatest common factor of two whole numbers (up to 20). AND **6.NS.4a2** Use a factor and distributive property to rewrite the sum of two whole numbers (up to 50). Models may be used.	**6.NS.4b** Identify factors of whole numbers (up to 50).	**6.NS.4c** Identify factors of whole numbers (up to 20).
Apply and extend previous understandings of numbers to the system of rational numbers.			
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**6.NS.5a** Represent real-world problems involving integers (e.g., temperatures, elevations, and distances from a fixed point (map reading).	**6.NS.5b** Identify the opposites of real-world examples of integers (e.g., opposite of gaining 40 yards is losing 40 yards).	**6.NS.5c** Identify or explain positive or negative regions in real-world models (e.g., sea elevations, yardage on football field, thermometer, etc.).

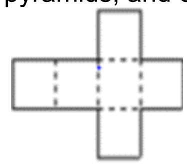
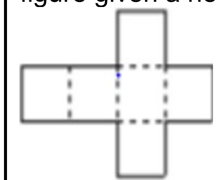
Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex			Least Complex
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., — (−3) = 3, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	**6.NS.6a1** Place 3 rational numbers on a number line. AND **6.NS.6a2** Identify the quadrants in terms of their sign; (+,+), (+,-), (-,-),(-,+).	**6.NS.6b1** Find an integer and its opposite on a number line. AND **6.NS.6b2** Explain the directionality of the x- and y-axis (horizontal vs. vertical).	**6.NS.6c1** Locate a given positive or negative number on a number line. AND **6.NS.6c2** Identify the quadrants of a coordinate plane as Quadrant I, Quadrant II, Quadrant III, Quadrant IV.
6.NS.7 Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7 °C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*	**6.NS.7a** On a number line, order rational numbers from smallest to largest (limit to 3 rational numbers).	**6.NS.7b** On a number line, order integers from smallest to largest (limit to 5 whole numbers).	**6.NS.7c** On a number line, order whole numbers from smallest to largest (limit to 3 whole numbers).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵		⟶	Least Complex
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**6.NS.8a** Plot whole number ordered pairs in a real-world example (e.g., mapping locations).	**6.NS.8b** Plot whole number ordered pairs.	**6.NS.8c** Identify the x- and y-axis, and plot an ordered pair in quadrant 1 of a coordinate plane.
Expressions and Equations			
Apply and extend previous understandings of arithmetic to algebraic expressions.			
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	**6.EE.1a** Write and evaluate numerical expressions involving exponents of squares and cubes only (e.g., $5^2 + 4^3$ evaluate). No variables should be used.	**6.EE.1b** Write and/or evaluate expressions with integers (e.g., a model of 10 apples and giving 2 away). No variables or exponents should be used.	**6.EE.1c** Identify a model that is equivalent to a numerical expression.
Apply and extend previous understandings of arithmetic to algebraic expressions.			
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 — y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*	**6.EE.2a** Given a context, student will write an algebraic expression for a context with 2 or 3 terms involving variables.	**6.EE.2b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) in simple expressions.	**6.EE.2c** Evaluate an algebraic expression with 2 to 3 terms. (The value of the variables to be substituted into the expression should be limited to whole numbers.)

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←		→	Least Complex
6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	**6.EE.3a** Identify equivalent algebraic expressions using distributive property (e.g., 2 (x + 3) is equivalent to 2x + 6).	**6.EE.3b** Identify equivalent algebraic expressions using commutative property (e.g., x + 3 is equivalent to 3 + x).	**6.EE.3c** Identify equivalent numerical expressions using the commutative property (e.g., 2 + 3 is equivalent to 3 + 2).
6.EE.4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	**6.EE.4a** Identify equivalent expressions (e.g., 2x + x is equivalent to 3x) with whole number coefficients.	**6.EE.4b** Identify equivalent expressions (limit to variables with no coefficient) (e.g., x + x is equivalent to 2x).	**6.EE.4c** Identify equivalent expressions (limit to whole number expressions) (e.g., 2 + 3 is equivalent to 5).
Reason about and solve one-variable equations and inequalities.			
6.EE.5 Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**6.EE.5a** Given an inequality statement in the form of x > c or x < c determines if a given value makes the inequality true. (For 3 > b, if b equals 5, is the inequality true?)	**6.EE.5b** Given a one-step equation and set of values for the variable, determine which value makes the equation true. (For 3x = 9, does x = 2, 3, 4, or 6 make the equation true?)	**6.EE.5c** Given a one-step equation and a value for the variable, determine if the value makes the equation true. (For 3x = 9, does x = 4 make the equation true?)
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	**6.EE.6a** Represent the missing information with a variable in a real-world problem.	**6.EE.6b** Identify the missing information in real-world problem.	**6.EE.6c** Identify a variable in an equation.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵			⟶ Least Complex
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all non-negative rational numbers.	**6.EE.7a** Using real-world contexts, write and solve one-step equations using whole numbers. Models may be used (e.g., Jim has several balloons. He gives 2 balloons to his brother and now has 5 balloons left. How many balloons did Jim have to begin with?).	**6.EE.7b** Solve a one-step equation using one of the 4 operations with whole numbers using models.	**6.EE.7c** Solve a one-step equation using addition and subtraction with whole numbers using models.
6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**6.EE.8a** Identify an inequality that is represented on a number line using a variable (e.g.,  $x < 8$)	**6.EE.8b** Identify an inequality that represents a real-world or mathematical problem (e.g., Jane is 23 years old. Sally is 19 years old. Which inequality correctly compares Jane and Sally's ages? $23 > 19$).	**6.EE.8c** Identify an inequality that compares two whole numbers using the >, <, and =.
Represent and analyze quantitative relationships between dependent and independent variables.			
6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	**6.EE.9a** Complete a table given a one-step equation and graph the coordinates.	**6.EE.9b** Find the outputs (dependent variable) given an equation and the inputs in table form. $y = 3 + x$ <table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>6</td></tr></table>	**6.EE.9c** Identify the input and output in a table.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟵		⟶	Least Complex
Geometry			
Solve real-world and mathematical problems involving area, surface area, and volume.			
6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.	**6.G.1a** Demonstrate that the area of a right triangle is $^1/_2$ × length × height (e.g., Two same right triangles combined make a rectangle, and the area of a triangle is half the area of the rectangle it can be composed into). Demonstrate these techniques in real-world and mathematics problems.	**6.G.1b** Demonstrate that the area of all rectangles is length × width (e.g., Multiply side lengths to find the area of rectangles with whole-number side lengths). Demonstrate these techniques in real-world and mathematics problems.	**6.G.1c** Find the area of rectangles with whole-number side lengths by counting unit squares. Demonstrate these techniques in real-world and mathematics problems.
6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	**6.G.2a** Recognize that the volume of a right rectangular prism can be found by multiplying the height by the area of the base (using whole numbers) (e.g., show that $V = \ell \cdot w \cdot h$ and $V = B \cdot h$). Limited to whole number edge lengths.	**6.G.2b** Demonstrate that unit cubes can be used to build figures that have volume, and determine the volume of a figure. Limit to whole number edge lengths.	**6.G.2c** Find the volume of a right rectangular prism (e.g., count the number of unit cubes it takes to fill a rectangular prism) (up to 25 cubes). Limit to whole number edge lengths.
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**6.G.3a** Find the length(s) of the side(s) of a polygon drawn in quadrant 1 of the coordinate plane.	**6.G.3b** Plot points of a polygon in quadrant 1 of the coordinate plane and identify the name of the shape.	**6.G.3c** Given a graph of plotted vertices, connect the vertices forming a polygon and identify the name of the shape that is created.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←			→ Least Complex
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**6.G.4a** Given a net and a three-dimensional figure, find the surface area of prisms, pyramids, and cubes. 	**6.G.4b** Identify a net given a three-dimensional figure or identify three-dimensional figure given a net. 	**6.G.4c** Identify cubes, rectangular prisms (e.g., cubes, rubber eraser, pyramids).
<u>Statistics and Probability</u>			
Develop understanding of statistical problem solving.			
6.SP.1 Develop statistical reasoning by using the GAISE model: **a**. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1) **b**. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)	**6.SP.1a** Recognize a statistical question.	**6.SP.1b** Identify when we might pose a statistical question.	**6.SP.1c** Ask questions about a statistical situation.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵			⟶ Least Complex
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**6.SP.2a** Given a data display, answer a statistical question about spread.	**6.SP.2b** Given a data display, answer a statistical question about center.	**6.SP.2c** Given a data display, answer a statistical question about shape.
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	**6.SP.3a1** Find the mean using models according to the idea of "fair share." Limit to 7 data points. AND **6.SP.3a2** Find the median of a data point. Limit to 7 data points.	**6.SP.3b** Find the median of a data set with an odd number of data points. Limit to 7 data points.	**6.SP.3c** Identify the mode for a set of data.
Summarize and describe distributions.			
6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3)	**6.SP.4a** Given pre-made axes, construct and interpret a histogram from a given or collected data set.	**6.SP.4b** Given pre-made axes, construct and analyze a line plot from a given or collected data set.	**6.SP.4c** Given pre-made axes, construct and analyze a bar graph from a given or collected data set.
6.SP.5 Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range), as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. **d.** Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	**6.SP.5a** Interpret information from a given or collected data set (e.g., Given a tally chart showing the number of pockets on students' clothes in a class. Find the average number of pockets and the range of the data).	**6.SP.5b** Interpret information from a given or collected data set (e.g., Given a tally chart showing the number of pets. Find the average number of pets).	**6.SP.5c** Interpret information from a given or collected data set (e.g., given a tally chart showing the favorite colors of the students in Joe's math class, determine which color was the most/least favorite).

GRADE 7

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex	⟷		Least Complex
Ratio and Proportional Relationships			
Analyze proportional relationships and use them to solve real-world and mathematical problems.			
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $^1/_2$ mile in each $^1/_4$ hour, compute the unit rate as the complex fraction $^{(1/2)}/_{(1/4)}$ miles per hour; equivalently 2 miles per hour.*	**7.RP.1a** Given a model or pictures of a ratio, build the unit rate (e.g., given 12 pieces of candy for $3, find the unit rate).	**7.RP.1b** Given a model of a unit rate, build equivalent ratios (e.g., every 4 pieces of candy cost $1. Using candy and play $1 bills, build equivalent ratios.).	**7.RP.1c** Given models of equivalent ratios, identify the unit rate. Using candy and play $1 bills, the student is shown 6 candies for $2, 9 candies for $3, and 3 candies for $1. Identify the unit rate.
7.RP.2 Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	**7.RP.2a1** Given a graph of a proportion, find an ordered pair on the line. **7.RP.2a2** Using models, determine whether two quantities represent a proportion.	**7.RP.2b1** Given three ordered pairs that represents a proportional relationship, plot them on a coordinate grid and connect the line. **7.RP.2b2** Find a missing value in a ratio table. Students may use manipulatives to find the answer.	**7.RP.2c1** Identify if a graph represents a proportional relationship. **7.RP.2c2** Build a proportion with objects such as blocks and record the information in a table.
7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	**7.RP.3a** Find the percent of a number in real-world problem involving tax or gratuity.	**7.RP.3b** Find the percent of a number.	**7.RP.3c** Find 10%, 20%, and 30% of a number given a model of 100 units.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
The Number System			
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.			
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers.	**7.NS.1a** Add and subtract integers.	**7.NS.1b1** Recognize that the absolute value of an integer is how far it is from 0 on the number line (e.g., plot a number and its opposite on a number line and recognize that they are equidistant from zero). **7.NS.1b2** Add and subtract whole numbers using models.	**7.NS.1c1** Recognize that addition means move to the right and subtraction means move to the left on a number line. **7.NS.1c2** Add whole numbers using models.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1) (−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**7.NS.2a** Multiply and divide integers using models.	**7.NS.2b** Multiply and divide whole numbers using models.	**7.NS.2c** Multiply whole numbers using models.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	**7.NS.3a** Multiply fractions when solving real-world and mathematical problems using models.	**7.NS.3b** Add and subtract fractions with same/unlike denominator when solving real-world and mathematical problems using models.	**7.NS.3c** Add fractions with same denominator when solving real-world and mathematical problems using models.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex	⟵	⟶	Least Complex
Expressions and Equations			
Use properties of operations to generate equivalent expressions.			
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**7.EE.1a** Apply the order of operations to problems using whole numbers. Limit the number of terms to 4.	**7.EE.1b** Apply the first step of the order of operations to create an equivalent expression. Limit the number of terms to 3.	**7.EE.1c** Identify the first step to complete the order of operations (e.g., 2 (3 + 5) – 10 + 2, what is the first step? Add 3 + 5.) Limit the number of terms to 3.
7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p — 0.15p) is equivalent to (1 — 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.*	**7.EE.2a** Create an equivalent expression by giving one missing term (limit to addition, subtraction, and multiplication, using whole numbers) (e.g., 6 × 4 = 8 × ?).	**7.EE.2b** Create an equivalent expression by giving one missing term (limit to addition and subtraction using whole numbers) (e.g., 7 + 1 = 6 + ?).	**7.EE.2c** Identify equivalent expressions (limit to addition using whole numbers) (e.g., 5 + 2 = 6 + 1).
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.			
7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional $^{1}/_{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 $^{3}/_{4}$ inches long in the center of a door that is 27 $^{1}/_{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	**7.EE.3a** Solve one-step real-life and mathematical problems (limit to fractions) (e.g., the recipe for 12 cupcakes asks for $^{2}/_{3}$ cup of sugar. How many cups of sugar is needed if the recipe is doubled?).	**7.EE.3b** Solve one-step real-life and mathematical problems (limit to decimals) (e.g., Sue spends $2.35 on a notebook and $1.60 on a ruler. How much does Sue spend in all?).	**7.EE.3c** Solve one-step real-life and mathematical problems (limit to whole numbers) (e.g., Jim spends $3 on a pen and $2 on a pencil. How much does Jim spend in all?).

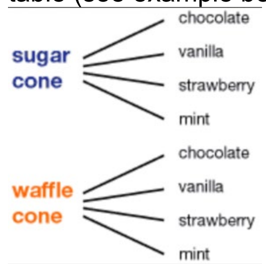
Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**7.EE.4a** Use variables to show and solve a real-world or mathematical problem (limit to two-step problems involving whole numbers and one variable) (e.g., Mary pays a $5 flat rate plus a $2 hourly rate for each hour, x, for parking. Mary has $15. Which equation should Mary use to calculate the total number of hours she can park? 2x + 5 = 15, 5x + 2 = 15, 2 + 5 + x = 15, or 15 + 5 + 2 = x).	**7.EE.4b** Use variables to solve a real-world or mathematical problem (limit to two-step problems and one variable) (e.g., Mary pays a $5 flat rate plus a $2 hourly rate for each hour, x, for parking. Mary has $15 is represented by $2x + 5 = 15$; solve for x.).	**7.EE.4c** Use variables to show a real-world or mathematical problem (limit to one-step problems involving whole numbers and one variable) (e.g., Mary has $15. She buys a bag of apples for $4. Which equation shows how much money, x, Mary has left? Key: x = 15 — 4).
<u>Geometry</u>			
Draw, construct, and describe geometrical figures and describe the relationships between them.			
7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **b.** Represent proportional relationships within and between similar figures.	**7.G.1a** Solve problems involving scaled drawings of figures (e.g., if a triangle is drawn on a grid, what will be the length of one of the sides if the triangle is increased by a factor of 2?).	**7.G.1b** Identify similar geometric figures on a grid (e.g., which shape is twice the size of another shape?).	**7.G.1c** Identify same size/same shape polygons drawn on a grid (e.g., square, rectangles, quadrilaterals, isosceles triangles, right triangles, scalene triangles, and obtuse triangles).

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷			Least Complex
7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions, noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.	**7.G.2a** Analyze special quadrilaterals and triangles **a.** by the measure of the angles (acute, obtuse, and right) by the measures of their side lengths (isosceles, equilateral, and scalene triangles; parallelogram, rhombus, and trapezoid).	**7.G.2b** Identify and recognize special quadrilaterals or triangles by using parallel and perpendicular sides.	**7.G.2c** Identify the type of angles in a triangle and the angles in a special quadrilateral.
7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**7.G.3a** Using models, identify two-dimensional shapes that result from slicing a three-dimensional figure (limit to prisms and horizontal and vertical cuts).	**7.G.3b** Identify the shape of two-dimensional faces of three dimensional figures (limit to rectangular prisms and cubes).	**7.G.3c** Identify, by naming, two- and three-dimensional figures (manipulatives can be used).
Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.			
7.G.4 Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.	**7.G.4a1** Apply the formula for finding the area of a circle when given the radius. **7.G.4a2** Measure diameters and circumference of various circles to show the relationship is close to 3.14.	**7.G.4b** Identify the attributes of a circle (radius, diameter, circumference, and center).	**7.G.4c1** Identify circles in the environment.
7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**7.G.5a** Identify unknown angles and solve problems when using facts about adjacent and vertical angles using visual models.	**7.G.5b** Classify angles as supplementary, complementary, vertical, or adjacent using visual models.	**7.G.5c** Sort angles by type using visual models (right, acute, obtuse, straight).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex	⟵	⟶	Least Complex
7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**7.G.6a1** Solve real-world problems involving surface area of a prism, cube, and pyramid. Use whole number edge lengths. **7.G.6a2** Solve real-world problems involving finding the volume of a right prism or cube. Use whole number edge lengths.	**7.G.6b** Solve real-world problems involving the area of figures involving rectangles and right triangles (manipulatives can be used).	**7.G.6c** Solve real-world problems involving perimeter (manipulatives can be used).
Statistics and Probability			
Use sampling to draw conclusions about a population.			
7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. **a.** Differentiate between a sample and a population. **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.	**7.SP.1a** Differentiate between a sample and population.	**7.SP.1b** Understand that a sample is only part of a population.	**7.SP.1c** Recognize that everyone in an area (such as classroom) is a population.

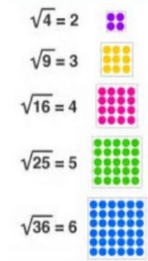
Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Broaden understanding of statistical problem solving.			
7.SP.2 Broaden statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)	**7.SP.2a1** Formulate statistical questions that include simple comparisons. AND **7.SP.2a2** Collect data to answer statistical questions.	**7.SP.2b** Given two questions, identify which question is statistical (anticipates variability). *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1).	**7.SP.2c** Identify the 4 steps of the GAISE model.
Summarize and describe distributions representing one population and draw informal comparisons between two populations.			
7.SP.3 Describe and analyze distributions. **a.** Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. **b.** Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.*	**7.SP.3a** Answer simple questions given two data displays (e.g., which data set has more people?).	**7.SP.3b** Find the mean and median of a data set. Limit to 7 data points.	**7.SP.3c** Answer questions given a graph (e.g., given a histogram of student's heights, which range of heights did most students fall into?).
7.SP.4 [Deleted standard]			

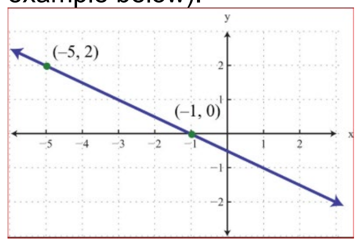
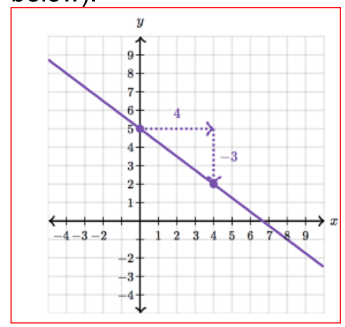
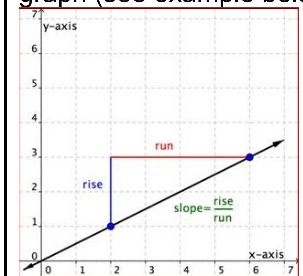
Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around $^{1}/_{2}$ indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.	**7.SP.5a** Given an outcome in a real-life event or situation, such as a game, determine if an event is impossible, likely, unlikely, or certain.	**7.SP.5b** Given an outcome in a real-life event or situation, such as a game, determine if an event is impossible, likely, or unlikely.	**7.SP.5c** Given an outcome in a real-life event or situation, such as a game, determine if an event is possible or impossible.
7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	**7.SP.6a** Approximate the probability of an event occurring as likely, unlikely, certain, or impossible based on possible outcomes using a model.	**7.SP.6b** Find the experimental probability of an event occurring after collecting data using a model.	**7.SP.6c** Collect data on the probability of an event (e.g. rolling dice, spinning a spinner, or drawing marbles).
7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	**7.SP.7a** Compare the probabilities of an event occurring (e.g., probability of landing on heads when flipping a coin; likelihood of landing on a certain area on a three section spinner).	**7.SP.7b** Use a probability model/graphic organizer to record data from a probability experiment (e.g., occurrence of heads or tails in a coin flip).	**7.SP.7c** Make prediction of the probability of an event occurring (e.g., probability of landing on heads when flipping a coin) using models.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex			Least Complex
7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **b.** Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which composes the event. **c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least four donors to find one with type A blood?*	**7.SP.8a** Find the number of outcomes of a compound events using a simple tree diagram, organized list, or table (see example below). 	**7.SP.8b** Complete a simple tree diagram, organized list, or table (see example blow). 	**7.SP.8c** Find the probability of a simple event (e.g., probability of landing on heads when flipping a coin).

GRADE 8

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
The Number System			
Know that there are numbers that are not rational, and approximate them by rational numbers.			
8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.	**8.NS.1a** Identify whether numbers are rational or irrational numbers.	**8.NS.1b** Identify whether numbers in decimal form are repeating or non-repeating decimals.	**8.NS.1c** Identify whether numbers are in the form of whole numbers, fractions or decimals.
8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., $\pi 2$. *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**8.NS.2a** Estimate which point on a number line a decimal (up to hundredths) is closest to (e.g., given a number line in increments of $^1/_{10}$, identify which point the decimal 4.13 would be closest to).	**8.NS.2b** Round decimals to the nearest whole number or tenths and identify the corresponding points on a number line.	**8.NS.2c** Identify the whole number points on a number line.
Expressions and Equations			
Work with radicals and integer exponents.			
8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = {}^1/_{3^3} = {}^1/_{27}$.*	**8.EE.1a** Apply properties of integer exponents to generate equivalent numerical expressions (e.g., $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4^5$).	**8.EE.1b** Identify equivalent numerical expressions with integer exponents; limit exponents to 1-6 (e.g., $3^4 = 3 \times 3 \times 3 \times 3$).	**8.EE.1c** Identify equivalent numerical expressions with integer exponents—limit to exponents 1-3 (e.g., $10^2 = 10 \times 10$).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex	←	→	Least Complex
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that it is irrational.	**8.EE.2a** Construct a perfect square up to 25 (e.g., 5 squared is 25). 	**8.EE.2b** Create a representation of a perfect square.	**8.EE.2c** Select the perfect square, given a model.
8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8; and the population of the world as 7×10^9; and determine that the world population is more than 20 times larger.*	**8.EE.3a** Identify equivalent expressions of numbers expressed in the form of a single digit times a whole number power of 10 (limit exponent to 1-10) (e.g., What is 3×10^3? Answer: 3000 or $3 \times 10 \times 10 \times 10$).	**8.EE.3b** Identify equivalent expressions of multiples of 10 using exponents (limit exponent to 1-10) (e.g., What is 10^7? Answer: $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$).	**8.EE.3c** Identify equivalent expressions of multiples of 10 using exponents (limit exponent to 1-5) (e.g., What is 10^2? Answer: 10×10).
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.	**8.EE.4a** Given a real-world context, write a number in scientific notation that best represents the situation.	**8.EE.4b** Given a real-world context and a selection of numbers written in scientific notation, select the quantity that best represents the situation.	**8.EE.4c** Interpret scientific notation that has been generated by technology.
Understand the connections between proportional relationships, lines, and linear equations.			
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**8.EE.5a** Identify the slope (unit rate) of a line of a proportional graph represented on a grid that has scales of 1.	**8.EE.5b** Graph a simple proportion with 3 coordinates.	**8.EE.5c** Identify if a graph represents a proportional relationship.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex	⟷		Least Complex
8.EE.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.	**8.EE.6a** Identify the slope of a line using a graph (see example below). 	**8.EE.6b** Identify the slope of a line using a graph in the first quadrant (see example below). 	**8.EE.6c** Identify the slope of a line using a graph in the first quadrant when the rise and run are shown on the graph (see example below). 
Analyze and solve linear equations and pairs of simultaneous linear equations.			
8.EE.7 Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**8.EE.7a** Solve a 1-step linear equation (e.g., y + 3 = 5).	**8.EE.7b** Identify the operation needed to solve a given 1-step linear equation (the inverse operation).	**8.EE.7c** When given a visual model, the students will correctly identify the missing variable from given choices (select from no more than 3, e.g., given the equation 4 + x = 2, identify that the variable is x).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟵		⟶	Least Complex
8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically. **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (Limit solutions to those that can be addressed by graphing.)	**8.EE.8a** Identify the coordinate at which two lines intersect.	**8.EE.8b** Locate the point where two lines intersect.	**8.EE.8c** Determine whether two lines intersect.
Functions			
Define, evaluate, and compare functions.			
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in grade 8.	**8.F.1a** Determine if a relation is a function when given in table form.	**8.F.1b** Determine if a relation is a function when given in graph form.	**8.F.1c** Identify the inputs and outputs of a function given in table form.
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*	**8.F.2a** Compare functions represented in the same form.	**8.F.2b** Classify graphs of functions as linear or non-linear. 	**8.F.2c** Determine whether a line is increasing, decreasing, or flat (zero slope).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	**8.F.3a** Match a function to its graph.	**8.F.3b** Determine whether a function is linear or non-linear given the equation or graph.	**8.F.3c** Determine whether the slope of the function is positive, negative or zero.
Use functions to model relationships between quantities.			
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**8.F.4a** Graph a linear function on a grid with a scale of 1.	**8.F.4b** Given a graph representing a linear equation, identify the slope and y-intercept.	**8.F.4c** Identify two points on a linear graph.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**8.F.5a** Tell a story using the qualitative features of a function.	**8.F.5b** Identify if a function in graph form is linear or nonlinear.	**8.F.5c** Identify if a function in graph form is increasing or decreasing or flat.
Geometry			
Understand congruence and similarity using physical models, transparencies, or geometry software.			
8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines.	**8.G.1a** Show experimentally (e.g., by measuring, overlapping figures, etc.) that congruent shapes have the same angle measures and side lengths.	**8.G.1b** Identify corresponding parts (angles and sides) on congruent shapes.	**8.G.1c** Identify congruent line segments and/or congruent angles.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them (include examples both with and without coordinates).	**8.G.2a** Determine the sequence of transformations (rotation, reflection, translation) that will make a figure congruent to another limit to two transformations in the sequence.	**8.G.2b** Determine whether a rotation, a reflection or a translation is needed to show whether one figure is congruent to another (limit to 1 transformation).	**8.G.2c** Determine the direction and how many units a figure must be translated (shifted) to be congruent to another on a coordinate plane (e.g., 3 units to the right).
8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	**8.G.3a** Compare the effects of dilations, translations, rotations and reflections (with or without coordinate grids). And/or perform a translation, rotations, and reflection on a grid.	**8.G.3b** Recognize the effect of rotation (turn), reflection (flip), and translation (slide).	**8.G.3c** Demonstrate concepts of translation (up, down, right, left) with manipulatives.
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them (include examples both with and without coordinates).	**8.G.4a** Perform a dilation on a figure using a scale factor of $^1/_2$, 2, or 3.	**8.G.4b** Recognize the effects of dilation on a two-dimensional figure (with or without coordinate grids). For example, recognize that applying a scale factor greater than one creates a bigger image and a scale factor less than one creates a smaller image.	**8.G.4c** Recognize and identify similar shapes and congruent figures (with or without coordinate grids). For example, if given a sheet of pictures of turtles or stars, recognize that the stretched images are not similar.
8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**8.G.5a** Given a pair of parallel lines cut by a transversal, identify corresponding angles, alternate interior angles, alternate exterior angles, supplementary angles, and vertical angles.	**8.G.5b** Know the sum of the interior angles of a triangle equals 180 degrees.	**8.G.5c** Identify triangles, parallel lines, perpendicular lines, and intersecting lines.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Understand and apply the Pythagorean Theorem.			
8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.	**8.G.6a** Know that triangles with the side lengths of 3, 4, 5, 6, 8, and 10 are right triangles.	**8.G.6b** Identify a right triangle when drawn on a coordinate plane.	**8.G.6c** Identify right triangles from a group of a variety of triangles.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	**8.G.7a** Find the length of the hypotenuse of a right triangle when given the lengths of the legs.	**8.G.7b** Place the numbers into the Pythagorean Theorem when given legs and hypotenuse of a right triangle.	**8.G.7c** Identify parts of a right triangle (hypotenuse, legs).
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**8.G.8a** Find the length of vertical and horizontal lines drawn on the coordinate grid.	**8.G.8b** Identify vertical and horizontal lines of a triangle drawn on the coordinate grid.	**8.G.8c** Identify vertices of a triangle on the coordinate grid.
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.			
8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.	**8.G.9a** Solve real-world and mathematical problems involving volume of cylinders.	**8.G.9b** Match the given formula for volume to cones, cylinders and spheres.	**8.G.9c** Find cones, cylinders, and spheres in the environment.
Statistics and Probability			
Investigate patterns of association in bivariate data.			
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)	**8.SP.1a** Construct a scatter plot for bivariate data using no more than 10 data points.	**8.SP.1b** Identify if a scatter plot has linear or nonlinear association.	**8.SP.1c** Identify if the pattern for a scatter plot is increasing or decreasing.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←			→ Least Complex
8.SP.2 Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)	**8.SP.2a** Determine which line most closely represents the line of best fit for a given scatterplot.	**8.SP.2b** Determine whether patterns on a scatter plot are positive, negative, or have no correlation.	**8.SP.2c** Determine whether a linear graph is increasing, decreasing, or flat.
8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4)	**8.SP.3a** Given a line of best fit, make a prediction.	**8.SP.3b** Given several lines on a scatterplot, identify which line most closely represents the line of best fit.	**8.SP.3c** Determine whether the line of best fit should have a positive, negative, or zero slope.
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**8.SP.4a** Identify an inside missing value in a table of values using the pattern.	**8.SP.4b** Find the totals in a two-way frequency table given the sums (outside values in the table).	**8.SP.4c** Identify the total population in a two-way frequency table (lower right-hand box).

Learning Standards for High School

*Additional mathematics standards for High School that represent complex numbers on the complex plane in rectangular and polar form (including real and Imaginary numbers) and are indicated by a + symbol in the Ohio's New Learning Standards are not included in the extended standards since they are not considered common mathematics curriculum for all college and career ready students.

NUMBERS AND QUANTITY

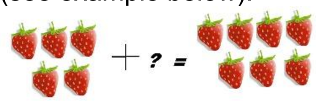
Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟵		⟶	Least Complex
The Real Number System			
Extend the properties of exponents to rational exponents.			
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**N.RN.1a** Identify equivalent expression with exponents (e.g. $3 \times 3 \times 3 \times 3$ is 3^4).	**N.RN.1b** Identify equivalent expression with exponents (limit to squares) (e.g. 3×3 is 3^2).	**N.RN.1c** Identify a number with an exponent.
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**N.RN.2a** Identify equivalent expression with exponents and radicals (e.g. $3 \times 3 \times 3 \times 3$ is 3^4).	**N.RN.2b** Identify equivalent expression with exponents and radicals (e.g. 3×3 is 3^2).	**N.RN.2c** Identify a number with a radical.
Use properties of rational and irrational numbers.			
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**N.RN.3a** Recognize the effects of multiplying and dividing with negative numbers (e.g., $-2 \times -4 = 8$).	**N.RN.3b** Recognize that the absolute value of a rational number is how far it is from 0 on the number line (e.g., plot a number and its opposite on a number line and recognize that they are equidistant from zero).	**N.RN.3c** Recognize that addition means move to the right and subtraction means move to the left on a number line.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ↔ Least Complex			
Quantities			
Reason quantitatively and use units to solve problems.			
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**N.Q.1a** Solve real-world problems involving positive and negative numbers (e.g., temperatures, elevations, and distance from a fixed point (map reading)).	**N.Q.1b** Solve problems involving positive and negative numbers using a number line (e.g., temperatures, distances from a fixed point).	**N.Q.1c** Locate a given positive or negative number on a number line.
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.	**N.Q.2a** Identify the appropriate unit of measure for volume.	**N.Q.2b** Identify the appropriate unit of measure for length.	**N.Q.2c** Identify units of measure.
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**N.Q.3a** Identify the appropriate unit of measure for volume.	**N.Q.3b** Identify the appropriate unit of measure for length.	**N.Q.3c** Identify units of measure.
The Complex Number System			
Perform arithmetic operations with complex numbers.			
N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a+bi$ with a and b real.	**N.CN.1a** Describe real and complex numbers.	**N.CN.1b** Given a set of numbers, compare the real and complex numbers.	**N.CN.1c** Given a set of numbers, label the real and complex numbers.
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**N.CN.2a** Add and subtract complex numbers.	**N.CN.2b** Identify a complex number.	**N.CN.2c** Identify an imaginary number.
Use complex numbers in polynomial identities and equations.			
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	**N.CN.7a** Solve a quadratic equation with real solutions.	**N.CN.7b** Given a graph of a quadratic equation, match the correct equation.	**N.CN.7c** Identify a coefficient.

ALGEBRA

Most Complex ⟷ Least Complex			
Interpret the structure of expressions.			
	A.SSE.1a Represent a real-world situation with an expression, both numerals and variables. Recognize parts of the expression in the real-world situation.	**A.SSE.1b** Represent a real-world situation with a numeric expression. Recognize parts of the expression in the real-world situation.	**A.SSE.1c** Represent a real-world situation with a model using concrete objects.
	A.SSE.2a Simplify expressions involving variables (e.g., $(2 (x + 4) = 2x + 8)$).	**A.SSE.2b** Identify the equivalent numeric expression (e.g., $7 + 5 = 5 + 7$).	**A.SSE.2c** Identify equivalent expressions with whole numbers less than 10 using concrete objects (e.g., objects, dots, etc.).
Write expressions in equivalent forms to solve problems.			
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **a.** Factor a quadratic expression to reveal the zeros of the function it defines. (A1, M2) **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A1, M2) **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.*	**A.SSE.3a** Apply properties of integer exponents to generate equivalent variable expressions (e.g., $b^2 \times b^4 = b^6$).	**A.SSE.3b** Apply properties of integer exponents to generate equivalent numerical expressions (e.g., $5^2 \times 5^4 = 5^6$).	**A.SSE.3c** Interpret numerical expressions with exponents (e.g., 5^4 means $5 \times 5 \times 5 \times 5$).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex	←	→	Least Complex
Arithmetic with Polynomials and Rational Expression Standards			
Perform arithmetic operations on polynomials.			
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) **b.** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)	**A.APR.1a** Add and subtract linear and/or quadratic polynomials. Models may be used.	**A.APR.1b** Add and subtract linear polynomials. Models may be used.	**A.APR.1c** Add linear polynomials. Models may be used.
Understand the relationship between zeros and factors of polynomials.			
A.APR.2 Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	**A.APR.2a** Multiply two binomials.	**A.APR.2b** Multiply a variable by a binomial.	**A.APR.2c** Identify a polynomial (binomials only).
A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.	**A.APR.3a** Find the zeros of a polynomial when the polynomial is factored (e.g., x^2 — 9 = 0 and $x^2 = 3x = 2 = 0$).	**A.APR.3b** Identify a polynomial (trinomial) (e.g., $x^2 = 3x = 2 = 0$).	**A.APR.3c** Identify a polynomial (binomial) (e.g., x^2 — 9 = 0).
Rewrite rational expressions.			
A.APR.6 Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**A.APR.6a** Identify a rational expression (e.g., $^{6x}/_3 = 2x$).	**A.APR.6b** Rewrite expressions in different forms (e.g., $x^2 + 1 = (x * x) + 1$).	**A.APR.6c** Given a visual model, identify an expression (e.g. $2 * 2 * 2 = 2^3$).

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Creating Equations Standards			
Create equations that describe numbers or relationships.			
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **c.** Extend to include more complicated function situations with the option to solve with technology. (A2, M3)	**A.CED.1a** Represent and solve a real-world situation with a two-step linear equation or inequality.	**A.CED.1b** Represent and solve a real-world problem with a one-step linear equation or inequality (e.g., Abby has $5, and she wants to buy a T-shirt for $8. How much more money does she need? Key: 5 + x = 8).	**A.CED.1c** Represent a real-world problem with a linear equation, using concrete objects, models and pictures (see example below). 
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **c.** Extend to include more complicated function situations with the option to graph with technology. (A2, M3)	**A.CED.2a** Create an equation with two variables to represent a linear relationship between quantities in a given context (e.g., y = 2x + 4).	**A.CED.2b** Using a two-variable equation describing a real-world situation, given the value of one variable, find and interpret the value of the other variable (e.g., Sally starts with $4 and gets an allowance of $2 each week. After x weeks, she has y = 2x + 4 dollars. When x = 3, find y and interpret the result).	**A.CED.2c** Identify the meaning of each number and/or variable in a given two-variable equation that describe a real-world situation (e.g., Sally starts with $4 and gets an allowance of $2 each week. After x weeks she has y = 2x + 4 dollars. What does 4 represent? What does x represent?).
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (A1, M1) **a.** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)	**A.CED.3a** Represent a constraint with an equation or inequality in two variables (e.g., x + y ≤ 8, describing the number of boys and girls in an 8-passenger van). (e.g., Abby has $15 to spend on a snack and two matching T-shirts. If she spends $3 on the snack, what is the maximum price of each T-	**A.CED.3b** Create a one-variable constraint using an inequality (e.g., x ≤ 6).	**A.CED.3c** Demonstrate a constraint using words or models (e.g., how many students can fit at this table?).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵			⟶ Least Complex
	shirt? Key: $x \le 5$).		
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **a.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle* $A = (\pi)r^2$ *to highlight radius r.* (A1) **b.** Focus on formulas in which the variable of interest is linear. *For example, rearrange Ohm's law V = IR to highlight resistance R.* (M1) **c.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange the formula for the area of a circle* $A = (\pi)r^2$ *to highlight radius r.* (M2) **d.** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)	**A.CED.4a** Rearrange a one-step formula to highlight a quantity (e.g., use the formula $a=lw$ to highlight the length of the rectangle by rearranging it to $l={}^{a}/_{w}$).	**A.CED.4b** Rearrange a one-step equation to solve for a variable (e.g., solve for x: $y = 2 + x$, ${}^{y}/_{2}=x$).	**A.CED.4c** Match a formula to a given situation (e.g., recognize that $a=lw$ is the formula for the area of a rectangle)
<u>Reasoning with Equations and Inequalities Standards</u>			
Understand solving equations as a process of reasoning and explain the reasoning.			
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**A.REI.1a** Order a given sequence of steps to solve an equation (e.g., $2x + 5 = 13$). Solve two-step equations with integer coefficients and solutions, explaining the steps.	**A.REI.1b** Determine a step needed to solve a two-step equation.	**A.REI.1c** Determine the step needed to solve a one-step equation (e.g., to solve $x + 5 = 13$, subtract 5 from both sides).
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**A.REI.2a** Solve linear equations with more than one step.	**A.REI.2b** Solve 1-step linear equations.	**A.REI.2c** Solve for the missing number within a given number sentence involving addition or subtraction of numbers less than 10.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟵		⟶	Least Complex
	Solve equations and inequalities in one variable.		
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**A.REI.3a** Solve a two- or three-step linear equation in one variable. Models may be used.	**A.REI.3b** Solve a one-step linear equation in one variable. Models may be used.	**A.REI.3c** Given a linear equation in one-variable and a list of possible solutions, identify the solution of the equation.
A.REI.4 Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x — p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.	**A.REI.4a** Identify or create perfect squares (e.g., square root of 25 = 5).	**A.REI.4b** Identify equivalent expressions that are cubes (e.g., $m^3 = m \times m \times m$).	**A.REI.4c** Identify equivalent expressions that are squared (e.g., $m^2 = m \times m$).
	Solve systems of equations.		
A.REI.6 Solve systems of linear equations algebraically and graphically. **a.** Limit to pairs of linear equations in two variables. (A1, M1) **b.** Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)	**A.REI.6a** Identify the coordinate at which two lines intersect.	**A.REI.6b** Locate the point on the graph at which two lines intersect.	**A.REI.6c** Identify whether two lines intersect.
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	**A.REI.7a** Locate the coordinate of the point(s) at which a line intersects a quadratic function (e.g., at which two coordinates does the line intersect the parabola?).	**A.REI.7b** Locate the point(s) on the graph at which a line intersects a quadratic function (e.g., identify on the graph where the line intersects the parabola).	**A.REI.7c** Identify whether a line intersects a quadratic function (e.g., does the line intersect the parabola at one or two points? Does the line intersect the parabola?).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵		⟶	Least Complex
Represent and solve equations and inequalities graphically.			
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	**A.REI.10a** Given a graph and an equation, fill out three points on a corresponding table of values.	**A.REI.10b** Given a table of values, graph the line on the coordinate plane.	**A.REI.10** Identify a point on a line on a coordinate plane.
A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, making tables of values, or finding successive approximations).	**A.REI.11a** Locate the coordinate at which two lines intersect. Using the x coordinate of the intersection point, substitute it back into the original equation to show that it is a solution of the equation.	**A.REI.11b** Locate the coordinate point on the graph at which two lines intersect.	**A.REI.11c** Identify whether two lines intersect.
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**A.REI.12a** Given a graph of an inequality including the shaded region, identify three points that make the inequality true.	**A.REI.12b** Identify on a graph of a line ≤, ≥ is represented by a solid line; and < and > are represented by a dotted line.	**A.REI.12c** Identify the graph of a linear inequality has a shaded region.

FUNCTIONS

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Interpreting Functions Standards			
Understand the concept of a function, and use function notation.			
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.	**F.IF.1a** Determine if a relation is a function.	**F.IF.1b** Complete an input-output table when given the function rule and values.	**F.IF.1c** Identify the input or output of a function given in table form.
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	**F.IF.2a** Given a linear equation using function notation, complete a table of values.	**F.IF.2b** Represent an equation in y= form with f(x).	**F.IF.2c** Understand that f(x)=y.
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1,\ f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.	**F.IF.3a** Given a sequence, determine the functional rule.	**F.IF.3b** Predict the next three terms in an arithmetic or geometric sequence (e.g., 3, 6, 9 …).	**F.IF.3c** Given a rule,identify the common ratio or common difference in a sequence.
Interpret functions that arise in applications in terms of the context.			
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (A2, M3) **a.** Focus on linear and exponential functions. (M1) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**F.IF.4a** Given a function made up of several linear functions, determine where the function is increasing, decreasing, or flat	**F.IF.4b** Given a graph of a linear equation, identify the y and/or x intercept.	**F.IF.4c** Determine whether the linear function is increasing, decreasing, or flat.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex			Least Complex
F.IF.5 Relate the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **a.** Focus on linear and exponential functions. (M1) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)	**F.IF.5a** Given the graph represented by a linear function, determine the domain.	**F.IF.5b** Given a context of a linear equation, describe the domain.	**F.IF.5c** Given a table, state the input values.
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (A2, M3)	**F.IF.6a** Identify the slope of a line when the equation is written in slope intercept form.	**F.IF.6b** Identify the slope of a line when given in graph form.	**F.IF.6c** Determine whether a slope is present on a given visual graph.

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex ⟷			Least Complex
Analyze functions using different representations.			
F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **a.** Graph linear functions and indicate intercepts. (A1, M1) **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3)	**F.IF.7a1** Graph a linear function using a graph with a scale of 1. AND/OR **F.IF.7a2** Determine whether an ordered pair is a viable solution to a given linear function.	**F.IF.7b1** Determine the y intercept point for a linear graph. AND/OR **F.IF.7b2** Determine whether the line is increasing (going up), decreasing (going down), or flat.	**F.IF.7c1** Identify two point on a linear graph. AND/OR **F.IF.7c2** Classify graphs of functions as linear or non-linear.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵⟶			Least Complex
Analyze functions using different representations.			
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2) **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change*[G] *in functions such as* $y = (1.02)^t$*, and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3) **i.** Focus on exponential functions evaluated at integer inputs. (A1, M2)	**F.IF.8a** Identify equivalent expressions (e.g. $2x + 2x = 4x$, $x^2 * x^2 = x^4$).	**F.IF.8b** Identify equivalent expressions (limit to three terms) (e.g. $x + x + x = 3x$, $x * x * x = x^3$).	**F.IF.8c** Identify equivalent expressions (limit to two terms) (e.g. $x + x = 2x$).
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **a.** Focus on linear and exponential functions. (M1) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**F.IF.9a** Compare a function given in table form to another function given in graphical form. *For example, which one is increasing?*	**F.IF.9b** Match a function given in table form to its graph.	**F.IF.9c** Match a function given as a verbal description to its graph. *For example, which is an increasing function?*

Most Complex			Least Complex
Build a function that models a relationship between two quantities.			
	F.BF.1a Create a linear function that represents a linear relationship between quantities in a given context.	**F.BF.1b** Given a linear function that describes a real-world situation and given the value of one variable, find and interpret the value of the other variable.	**F.BF.1c** Identify the meaning of each number and/or variable in a linear function that describes a real-world situation.
	F.BF.2a Identify the rule for a pattern.	**F.BF.2b** Identify the next term in a pattern.	**F.BF.2c** Determine if a given set represents a pattern.
Build new functions from existing functions.			
F.BF.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) **a.** Focus on transformations of graphs of quadratic functions, except for *f*(*kx*. (A1, M2)	**F.BF.3a** Identify a line reflected over the y-axis on a coordinate grid.	**F.BF.3b** Identify a line reflected over the x-axis on a coordinate grid.	**F.BF.3c** Identify a line on a coordinate grid.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
F.BF.4 Find inverse functions. **a.** Informally determine the input of a function when the output is known. (A1, M1)	**F.BF.4a** Solve for x when y is given (e.g. y = x + 3; what is the value of x when y is 5?).	**F.BF.4b** Identify the input and output of a function.	**F.BF.4c** Identify a function.
Linear Quadratic and Exponential Models Standards			
Construct and compare linear, quadratic, and exponential models, and solve problems.			
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	**F.LE.1a** Identify a situation that represents a linear and/or exponential function.	**F.LE.1b** Identify the graph of a linear function and an exponential function.	**F.LE.1c** Identify a graph of a linear function.
F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).	**F.LE.2a** After creating a sequence, make a graph that represents that sequence.	**F.LE.2b** Create a geometric sequence of at least 5 numbers with models.	**F.LE.2c** Create an arithmetic sequence with a model.
F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2)	**F.LE.3a** Observe a situation that shows increasing and decreasing linear and exponential events. (e.g., doubling a penny every day gives you more money than receiving $100 a day for a month)	**F.LE.3b** Identify an exponential function.	**F.LE.3c** Identify if a linear function is increasing or decreasing.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where *a*, *c*, and *d* are numbers, and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	**F.LE.4a** Identify equivalent expressions with exponents.	**F.LE.4b** Identify equivalent expressions with exponents (limit to power 3 expressions) (e.g. Which is the same as m × m × m?).	**F.LE.4c** Identify equivalent expressions with exponents (limit to power 2 expressions) (e.g. Which is the same as m × m?).
Interpret expressions for functions in terms of the situation they model.			
F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	**F.LE.5a** Given a context, interpret the parameters of an exponential function (e.g., during x weeks, the existing number of fish in the pond has been doubled). This situation is modeled by the exponential function y = $100(2^x)$, where 100 is the initial number of fish in the pond, 2 is a growth factor, (2^x) is the number by which the initial number of fish, 100, is multiplied for every increase in x, and y is the total number of fish in the pond.	**F.LE.5b** Given a context, interpret the parameters of a linear function (e.g., Marsha has $10 already saved and saves an additional $5 a week for x number of weeks). This situation is modeled by a linear function f(x) = 5x +10, where 10 is the initial amount that has been saved, 5 is the weekly saving, 5x is the amount of money saved during x weeks, and f(x) is a total amount of money saved including the initial amount.	**F.LE.5c** Identify the constant in a linear or exponential equation. OR When given the graph of a function, identify the y intercept.
Trigonometric Functions Standards			
Extend the domain of trigonometric functions using the unit circle.			
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	**F.TF.1a** Identify the measure of a central angle on a circle when the measure of the arc is given.	**F.TF.1b** Identify the measure of an angle.	**F.TF.1c** Identify an angle.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**F.TF.2a** Identify the measure of a central angle on a circle when the measure of the arc is given.	**F.TF.2b** Identify the measure of an angle.	**F.TF.2c** Identify an angle.
Model periodic phenomena with trigonometric functions.			
F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**F.TF.5a** Identify the measure of a central angle on a circle when the measure of the arc is given.	**F.TF.5b** Identify the measure of an angle.	**F.TF.5c** Identify an angle.
Prove and apply trigonometric identities.			
F.TF.8 Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1, and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	**F.TF.8a** Find the hypotenuse when the length of the sides is given.	**F.TF.8b** Identify the parts of a right triangle.	**F.TF.8c** Identify a right triangle.

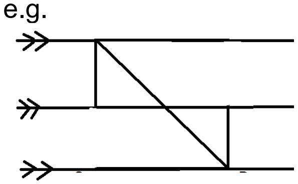
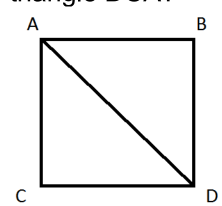
GEOMETRY

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ⟷			Least Complex
Congruence			
Experiment with transformations in the plane.			
G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.	**G.CO.1a** Identify points, lines, line segments, angles (right, acute, obtuse, and order by size), and perpendicular and parallel lines.	**G.CO.1b** Identify points, lines, line segments and angles (right, acute, obtuse, and order by size).	**G.CO.1c** Identify points, lines, and line segments, and order angles by size.
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.	**G.CO.2a** Demonstrate that a rotation (turn), a reflection (flip), or a translation (slide) maps a figure onto another.	**G.CO.2b** Identify whether a rotation (turn), a reflection (flip), or a translation (slide) can map a figure onto another.	**G.CO.2c** Match shapes in different orientations. (i.e., shapes = 2D)
G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.	**G.CO.3a** Show that two figures have symmetry on a coordinate plane.	**G.CO.3b** Identify figures that have line symmetry or rotational symmetry, using concrete objects or on a coordinate plane.	**G.CO.3c** Given visual models, determine which figures have line symmetry. (i.e., figure =3D)
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	**G.CO.4a** Identify that a translation requires a direction and distance; a rotation requires a center and angle; and a reflection requires a line.	**G.CO.4b** Identify whether a transformed figure is a "translation," "reflection," or "rotation."	**G.CO.4c** Identify whether a transformed figure is a "slide," "flip," or "turn."

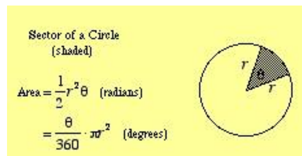
Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ↔			Least Complex
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**G.CO.5a** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.	**G.CO.5b** Given visuals or real-world items, demonstrate a rotation (turn), a reflection (flip), or a translation (slide).	**G.CO.5c** Match shapes in different orientations.
Understand congruence in terms of rigid motions.			
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent	**G.CO.6a** Identify a basic rigid motion (a rotation (turn), a reflection (flip), or a translation (slide) that maps one figure onto another. (Restrict to situations in which a single basic rigid motion suffices.)	**G.CO.6b** Show two figures are congruent by demonstrating that a rotation (turn), a reflection (flip), or a translation (slide) maps one onto the other.	**G.CO.6c** Match shapes to show congruence by placing one figure on top of the other.
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	**G.CO.7a** Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle on a coordinate plane. Limit to one transformation.	**G.CO.7b** Identify whether a rotation (turn), a reflection (flip), or a translation (slide) is required to show that a triangle is congruent to another triangle. Limit to one transformation.	**G.CO.7c** Match triangles in different orientations.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	**G.CO.8a** Determine whether two triangles are congruent using ASA, SAS, or SSS.	**G.CO.8b** Match corresponding parts (sides and angles) of congruent triangles.	**G.CO.8c** Match one corresponding part (side or angle) of two congruent triangles.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex	↔		Least Complex
Prove geometric theorems both formally and informally using a variety of methods.			
G.CO.9 Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	**G.CO.9a1** Identify a pair of vertical, complementary, supplementary, corresponding, alternative interior, or alternate exterior angles. **G.CO.9a2** Find a missing angle measure for situations involving vertical, complementary, supplementary, corresponding, alternative interior, and alternative exterior angles.	**G.CO.9b1** Given a pair of vertical angles and a missing angle measurement, find the missing angle measure. **G.CO.9b2** Bisect a line segment using a ruler, compass, technology, or other means and label the midpoint. **G.CO.9b3** Create a pair of perpendicular lines using a ruler, compass, technology or other means. Include the right-angle marking.	**G.CO.9c1** Identify vertical angles. **G.CO.9c2** Identify a set of perpendicular lines. **G.CO.9c3** Identify the midpoint of a line segment. Identify a right angle.
G.CO.10 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	**G.CO.10a** Determine the sum of the measures of the interior angles of a triangle. Identify congruent angles in isosceles and equilateral triangles.	**G.CO.10b** Identify right, equilateral, and isosceles triangles.	**G.CO.10c** Identify a triangle.
G.CO.11 Prove and apply theorems about parallelograms. Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	**G.CO.11a** Identify the congruent sides and angles of a parallelogram.	**G.CO.11b** Identify congruent sides on a parallelogram.	**G.CO.11c** Identify a rectangle and a parallelogram.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ↔			Least Complex
Make geometric constructions.			
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	**G.CO.12a** Construct a circle given a center and a radius. Copy a segment.	**G.CO.12b** Construct a line segment given its endpoints.	**G.CO.12c** Identify geometric tools (e.g., straightedge, protractor, and ruler) and their uses.
G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**G.CO.13a** Construct an equilateral triangle.	**G.CO.13b** Construct a circle given a center and point on the circle.	**G.CO.13c** Given three congruent line segments (or sticks), make an equilateral triangle.
Classify and analyze geometric figures.			
G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.	**G.CO.14a** Classify two-dimensional shapes based on their properties.	**G.CO.14b** Sort different types of quadrilaterals.	**G.CO.14c** Sort different types of triangles.
<u>Similarity Right Triangles and Trigonometry</u>			
Use complex numbers in polynomial identities and equations.			
G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.	**G.SRT.1a** Determine the dimensions of a figure after dilation.	**G.SRT.1b** Determine if a figure is bigger or smaller after dilation.	**G.SRT.1c** Compare 2 figures to determine if a dilation has occurred.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←→			Least Complex
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations, the meaning of similarity for triangles as the equality of all corresponding pairs of angles, and the proportionality of all corresponding pairs of sides.	**G.SRT.2a** Determine if figures are similar; describe or select why two figures are or are not similar.	**G.SRT.2b** Determine if two rectangles or triangles are similar.	**G.SRT.2c** Identify similar triangles.
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**G.SRT.3a** Identify similar triangles in different orientations.	**G.SRT.3b** Identify similar triangles.	**G.SRT.3c** Identify a triangle.
Prove and apply theorems both formally and informally involving similarity using a variety of methods.			
G.SRT.4 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.	**G.SRT.4a** Identify different types of triangles.	**G.SRT.4b** Identify parts of a right triangle.	**G.SRT.4c** Identify a right angle in the environment.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.	**G.SRT.5a** Identify if triangles are similar, or not, in a given geometric figure; e.g. 	**G.SRT.5b** Identify if triangles are similar or not in a decomposed polygon; e.g., is triangle ABD similar to triangle DCA? 	**G.SRT.5c** Identify similar triangles.
Define trigonometric ratios, and solve problems involving right triangles.			
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	**G.SRT.6a** Identify parts of a right triangle.	**G.SRT.6b** Identify right triangles.	**G.SRT.6c** Identify a triangle.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←			→ Least Complex
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	**G.SRT.7a** Identify parts of a right triangle.	**G.SRT.7b** Identify right triangles.	**G.SRT.7c** Identify an angle of a triangle.
G.SRT.8 Solve problems involving right triangles. **a.** Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2) **b.** Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems. (A2, M3)	**G.SRT.8a** Construct a right triangle on a coordinate plane and label the parts.	**G.SRT.8b** Identify the parts of a right triangle (right angle, legs, and hypotenuse).	**G.SRT.8c** Given an assortment of triangles, identify right triangles.
Circles			
Understand and apply theorems about circles.			
G.C.1 Prove that all circles are similar using transformational arguments.	**G.C.1a** Compare two circles and determine how to change one to make it the same as the other (e.g., circle A needs to be enlarged to match circle B).	**G.C.1b** Label the parts of a circle.	**G.C.1c** Locate circles in the environment.
G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs, and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	**G.C.2a** Use the radius of a circle to determine the length of the diameter and vice versa.	**G.C.2b** Identify parts of a circle (radius, diameter, circumference, chord, and arc).	**G.C.2c** Locate circles in the environment.
G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscrived in a circle.	**G.C.3a** Identify a circumscribed circle about a triangle	**G.C.3b** Identify a circle inscribed in a triangle.	**G.C.3c** Identify a circle.

<table>
<tr><th>Learning Standard</th><th>Complexity a</th><th>Complexity b</th><th>Complexity c</th></tr>
<tr><td>Most Complex</td><td></td><td></td><td>Least Complex</td></tr>
<tr><td colspan="4">Find arc lengths and areas of sectors of circles.</td></tr>
<tr><td>G.C.5 Find arc lengths and areas of sectors of circles.
a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.
b. Derive the formula for the area of a sector, and use it to solve problems.</td><td>G.C.5a Identify the central angle of a circle.
OR
Apply the formula to the area of a sector (e.g., area of a slice of pie).

</td><td>G.C.5b Identify the sector of a circle.</td><td>G.C.5c Identify the arc of a circle.</td></tr>
<tr><td>G.C.6 Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)</td><td>G.C.6a Identify the central angle of a circle.</td><td>G.C.6b Identify the sector of a circle.</td><td>G.C.6c Identify the arc of a circle.</td></tr>
<tr><td colspan="4"><u>Expressing Geometric Properties with Equations</u></td></tr>
<tr><td colspan="4">Translate between the geometric description and the equation for a conic section.</td></tr>
<tr><td>G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean theorem; complete the square to find the center and radius of a circle given by an equation.</td><td>G.GPE.1a Identify the diameter of a circle.</td><td>G.GPE.1b Identify the radius of a circle.</td><td>G.GPE.1c Identify a circle.</td></tr>
<tr><td colspan="4">Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.</td></tr>
<tr><td>G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle. (G, M2)</td><td>G.GPE.4a Find the perimeter of quadrilaterals drawn on a coordinate grid.</td><td>G.GPE.4b Identify shapes on a coordinate grid.</td><td>G.GPE.4c Identify special triangles, quadrilaterals, and circles.</td></tr>
</table>

Learning Standard	Complexity a	Complexity b	Complexity c
Most Complex			Least Complex
G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.	**G.GPE.5a** Describe the "rise and run" relationships between two perpendicular lines.	**G.GPE.5b** Identify the slopes of parallel and perpendicular lines in a coordinate grid.	**G.GPE.5c** Identify parallel and perpendicular lines in a coordinate grid.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**G.GPE.6a** Find the midpoint of a vertical or horizontal line on a coordinate grid.	**G.GPE.6b** Find the length of a vertical or horizontal line on a coordinate grid.	**G.GPE.6c** Identify points, lines, and line segments.
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	**G.GPE.7a** Find the area and perimeter of shapes given on a coordinate grid. (Restrict to shapes with sides that are vertical or horizontal line segments.)	**G.GPE.7b** Find the perimeter of shapes given on a coordinate grid. (Restrict to shapes with sides that are vertical or horizontal line segments.)	**G.GPE.7c** Identify shapes on a coordinate grid.
Geometric Measurement and Dimension			
Explain volume formulas, and use them to solve problems.			
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	**G.GMD.1a** Compare the volume of two objects with the same base but different heights and vice versa (e.g., Which cup can hold more water: the shorter or the taller cup; given the choice of different sized cubes, identify which would hold more).	**G.GMD.1b** Distinguish between objects that do and do not have volume.	**G.GMD.1c** Sort three-dimensional objects (cones, cylinders, spheres).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵		⟶	Least Complex
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	**G.GMD.3a** Compare the volume of two objects with the same base but different heights and vice versa (e.g., Which cup can hold more water: the shorter or the taller cup; given the choice of different sized cubes, identify which would hold more).	**G.GMD.3b** Distinguish between objects that do and do not have volume.	**G.GMD.3c** Sort three-dimensional objects (cones, cylinders, spheres, pyramids).
Visualize relationships between two-dimensional and three-dimensional objects.			
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**G.GMD.4a** Identify cross-sections of three- dimensional shapes.	**G.GMD.4b** Identify faces of three-dimensional shapes.	**G.GMD.4c** Identify two- and three-dimensional shapes.
Understand the relationships between length, area, and volume.			
G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.	**G.GMD.5a** Compare the volume of three-dimensional shapes.	**G.GMD.5b** Compare the area of shapes.	**G.GMD.5c** Compare similar shapes.
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.	**G.GMD.6a** Find the volume of two similar three-dimensional shapes.	**G.GMD.6b** Find the area of two similar shapes.	**G.GMD.6c** Identify similar shapes.

Learning Standard	***Complexity a***	***Complexity b***	***Complexity c***
Most Complex ←		→	Least Complex
Modeling with Geometry			
Apply geometric concepts in modeling situations.			
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.	**G.MG.1a** Connect the shape of real-world objects to two-dimensional and three-dimensional shapes (e.g., the trunk of a tree is cylindrical in shape; a car is cube in shape; the center of a sunflower is circular in shape; a bookshelf is rectangular prism in shape).	**G.MG.1b** Connect the shape of real-world objects to two-dimensional shapes (e.g., a window is rectangular in shape, a wheel is circular in shape, and a table can be of many different shapes).	**G.MG.1c** Connect two-dimensional shapes with real-world objects.
G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.	**G.MG.2a** Calculate and compare the densities of two datasets in the same modeling situation (e.g, Is the population density of Ohio or New York greater?).	**G.MG.2b** Calculate the density of a given situation.	**G.MG.2c** Given representations of density, identify the one with the greatest or least density (e.g., If 3 squares of the same size have different numbers of dots in them, which one has the greatest number of dots?).
G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.	**G.MG.3a** Compare the volume of real-world objects.	**G.MG.3b** Compare the area of real-world objects.	**G.MG.3c** Sort shapes that model a real-world object (e.g., a baseball is a sphere, a can of soup is a cylinder).

STATISTICS AND PROBABILITY

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵		⟶	Least Complex
Interpreting Categorical and Quantitative Data			
Summarize, represent, and interpret data on a single count or measurement variable.			
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.	**S.ID.1a** Collect data in real-world context to create a dot plot, histogram, or box plot to represent collected data.	**S.ID.1b** Create a dot plot, histogram, or a box plot to represent given or collected data.	**S.ID.1c** Match given data to a given dot plot, histogram, or box plot.
S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets.	**S.ID.2a** Compare mean, median, and mode of 2 or more given graphs or collected data sets.	**S.ID.2b** Compute mean, median, or mode of a given graph or collected data set involving numbers less than 100.	**S.ID.2c** Identify the median and mode of a graph or a given data set involving numbers less than 50.
S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**S.ID.3a** Interpret a dot plot.	**S.ID.3b** Interpret a histogram.	**S.ID.3c** Complete an incomplete dot plot, box plot, or histogram (e.g., adding missing labels and missing data points).
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**S.ID.4a** Organize given data into a normal distribution graph.	**S.ID.4b** Formulate the mean of given data for the normal distribution.	**S.ID.4c** Identify the normal distribution of given data.
Summarize, represent, and interpret data on two categorical and quantitative variables.			
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	**S.ID.5a** Determine the missing value in a two-way frequency table using the given context.	**S.ID.5b** Given a two-way frequency table, within a context, determine the missing value(s).	**S.ID.5c** Identify the "most" or "least" value in a two-way frequency table.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3) **b.** Informally assess the fit of a function by discussing residuals. (A2, M3) **c.** Fit a linear function for scatterplot that suggests a linear association. (A1, M1)	**S.ID.6a** Create a scatter plot to represent given or collected data and interpret the relation between the two variables as positive, negative, or no correlation.	**S.ID.6b** Create a scatter plot for a given data set. (Limited to 8 data points.)	**S.ID.6c** Interpret the relation between two variables in a scatter plot as positive, negative, or no correlation.
Interpret linear models.			
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	**S.ID.7a** Interpret in a real-world context a line of best fit with a given slope and *y*-intercept for a scatter plot.	**S.ID.7b** Identify the y-intercept and slope of a line of best fit for a scatter plot.	**S.ID.7c** Match a line graph with a given data set.
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	**S.ID.8a** Construct data plots to identify strong and weak correlations of given data.	**S.ID.8b** Identify the strongest and weakest correlations given visual representation of data.	**S.ID.8c** Identify the strongest correlation given a visual representation of data.
S.ID.9 Distinguish between correlation and causation.	**S.ID.9a** Describe real-world situations that illustrate correlation and/or causation (e.g., rain = umbrella).	**S.ID.9b** Name correlation in real-world example.	**S.ID.9c** Identify correlation and causation in real-world examples (e.g., shoe size vs. height).
<u>Making Inferences and Justifying Conclusions</u>			
Understand and evaluate random processes underlying statistical experiments.			
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**S.IC.1a** Determine if the given data could come from a specific data-generating device (spinner, coin, number cube).	**S.IC.1b** Determine the likelihood (likely, impossible, unlikely, and certain) of outcomes from a data-generating device.	**S.IC.c1** Determine the likelihood (certain or impossible) of an outcome from a data-generating device.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟵		⟶	Least Complex
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	**S.IC.2a** Understand a probability of 0 as impossible, a probability of 1 as certain, a probability near 0 as unlikely, near 1 as likely, and near $^1/_2$ as equally likely.	**S.IC.2b** Understand a probability near 0 as unlikely and near 1 as likely.	**S.IC.2c** Understand a probability near 0 as unlikely and near 1 as likely using a number line.
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.			
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	**S.IC.3a** Identify sample surveys, experiments, and observational studies.	**S.IC.3b** Identify a sample survey and an experiment.	**S.IC.3c** Identify a sample survey.
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.	**S.IC.5a** Compare data of a randomized experiments to determine outcome differences.	**S.IC.5b** Determine if a given treatment changed the outcome of an experiment.	**S.IC.5c** Match the given treatment that changed the outcome (e.g., bleach changed the stain, water did not).
S.IC.6 Evaluate reports based on data.	**S.IC.6a** Evaluate if data supports the claim/results. Evaluate given data to determine results.	**S.IC.6b** Determine if data supports the results/claim.	**S.IC.6c** Match the data to the given results.
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	**S.IC.4a** Estimate the mean of data given in a sample survey.	**S.IC.4b** Determine the mean of data given in a sample survey.	**S.IC.4c** Match the mean of data given in a sample survey.
Conditional Probability and The Rules of Probability			
Understand independence and conditional probability, and use them to interpret data.			
S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**S.CP.1a** Calculate the probability of given event.	**S.CP.1b** List all possible outcomes of an event.	**S.CP.1c** Choose the possible outcomes of an event (e.g., 4 possible colors spun on a 4-section spinner).

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	**S.CP.2a** Create a Venn Diagram given categorical data.	**S.CP.2b** Calculate probabilities based on a Venn Diagram.	**S.CP.2c** Arrange given data into a Venn Diagram (e.g., sports teams).
S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	**S.CP.3a** Calculate conditional probabilities of events from Venn Diagrams using the addition rule (e.g., the probability of students who like horror or comedy movies who also like pizza).	**S.CP.3b** Calculate conditional probabilities of events. (e.g., chance of drawing an ace or face card).	**S.CP.3c** Arrange given data into a Venn Diagram.
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.*	**S.CP.4a** Create a two-way frequency table when given data and calculate the probability of an event.	**S.CP.4b** Given a two-way frequency table, calculate the probability of an event.	**S.CP.4c** Complete a two-way frequency table given data.
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	**S.CP.5a** Given a real-world scenario, student will name the conditional probabilities and their effects.	**S.CP.5b** Given a real-world scenario, student will name the conditional probabilities.	**S.CP.5c** Given a real-world scenario, student will determine if the situation or event is conditional or independent.
Use the rules of probability to compute probabilities of compound events in a uniform probability model.			
S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	**S.CP.6a** Given a Venn Diagram or a table with "*a given* b" statement, identify "*a"* and "*b"*.	**S.CP.6b** Given a Venn Diagram or a table, distinguish dependent and independent events.	**S.CP.6c** Given a Venn Diagram or a table and data, correctly input missing data on the table.

Learning Standard	*Complexity a*	*Complexity b*	*Complexity c*
Most Complex ⟷			Least Complex
S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.	**S.CP.7a** Write the Addition Rule equation given a complete two-way table.	**S.CP.7b** Name/identify 2 missing condition variables and input into equation.	**S.CP.7c** Given a two-way table and the Addition Rule with missing condition, student will identify one missing variable.

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